VALUATION

A Critique of Total Cost of Equity

Why TCOE Results May Not Be Defensible

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In the controversy over the theoretical appropriateness of the Butler-Pinkerton Model (BPM), which its creators now refer to as The Butler Pinkerton Calculator, the focus has been almost exclusively on the Total Beta (Tβ) term. Specifically, is Total Beta a valid measure of investor risk, and can it be used as a substitute for (or, as some would suggest, an improvement upon) the CAPM-derived Beta (i.e., Market Beta or β)? A great deal has been written both for and against Total Beta and the BPM.

At the very heart of the controversy is a proposal by Peter J. Butler and Keith A. Pinkerton to employ Professor Aswath Damodaran's "relative volatility statistic" (e.g., Total Beta) in a new arrangement that purportedly would calculate the company-specific risk premium (CSRP) of any public proxy firm. As the debate evolved, Butler and Pinkerton became less emphatic that the model could be used to determine a CSRP and became much more concerned with the use of the BPM as a determinant of total risk and the total cost of equity (TCOE) for any guideline firm. The chief detractor of the BPM has been Larry J. Kasper, who has written a number of highly technical articles primarily aimed at discrediting Total Beta as a risk multiple.

In contrast, the purpose of this article will be to pragmatically examine the underlying concepts of Total Beta and TCOE. The goal here will be to first consider if these risk measurements are conceptually sound, and if they are, assess their overall usefulness in the everyday practice of business valuation. My findings, generally, are that Total Beta could provide another useful insight into the historical risk profile of a proxy firm, but that the use of the equity risk premium (ERP) in the TCOE formula is open to question. Market Beta, while undoubtedly theoretically sound, is fraught with practical *ex post* measurement problems.

Much of the efforts the BPM detractors have been expended in the argument that Total Beta is theoretically invalid as a substitute for Market Beta, and contravenes the rationale of Capital Market Theory. This is ironic because Market Beta and Total Beta eventually do converge. That is, because Total Beta = σ_i/σ_m[1] and Market Beta = $\rho(\sigma_i/\sigma_m)$,[2] and therefore, as $\rho \to 1$, then Total Beta = Market Beta and the TCOE = k_e (the traditional *ex ante* CAPM cost of equity).[3]

In order to come to a clearer understanding of the salient issues, a greater degree of precision is required when referencing Market Beta and the financial theory surrounding it as to whether one is speaking of the *ex ante* (forward looking) theoretical model vs. the *ex post* (using actual historical data) practical application. As Kasper has pointed out in a number of his writings, the original CAPM theory in William F. Sharpe's 1964 landmark paper was *ex ante*[4] and dealt with the *expected* returns of securities and their covariance with the market (or an efficient combination of risky assets instead of the market). In an *ex ante* context, at least according to the homogeneous expectations of theoretically well diversified investors, the non-systematic or idiosyncratic return to any efficient portfolio is nil. Diversification insulates these investors from any *expected* exposure to company-specific risk. In contrast, the *ex post* measurement of any

1 Where σ_i is the standard deviation (i.e. volatility) of the proxy firm's returns, and σ_m is the standard deviation of the market—or practically speaking, a Market Index that is selected to represent the market.

2 Where ρ is the correlation coefficient and is equal to the covariance of the returns on the proxy security and the Market Index divided by the product of the proxy volatility and the Market Index volatility.

3 Normally it would be very unusual to find a ρ as high as 1.0 (or 0.9 or even 0.8). The point is that, theoretically, the two Beta measures, for practical applications, do become the same at very high levels of ρ. This also suggests that Market Beta supporters would become increasingly dissatisfied with TCOE results as ρ descends.

4 The exception is in Section IV, the text describing Figure 8 where Sharpe points out that an *ex post* regression will consist of two components: the slope of the regression line is the systematic risk, and the standard error of the y predictions will represent the unsystematic component. "The relationship between Ri and Rg can be employed *ex ante* as a predictive model." That is, the slope of the *ex post* regression can be used to predict future systematic risk. At no point, however, does he suggest that the regressed standard error of the y terms can be used to predict future non-systematic risk—and this, of course, is because, *ex ante*, expected non-systematic risk would be fully diversified and, ergo, nil. William F. Sharpe, "Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk," *The Journal of Finance*, 1964, v19, pp. 425-442.

actual individual company returns are riddled with company-specific (non-systematic) price shifts.[5] Herein lies the greatest difficulty of using historical market data for the prediction of "true" Beta: It is impossible to segregate the historic stock price shifts into the constituent components of systematic vs. non-systematic. Sharpe said, "A security's 'true' historical Beta cannot be observed. All that can be done is to estimate its value."[6] And, "historical betas for individual securities are subject to great error and should be treated accordingly."[7]

MARKET ERP AND VOLATILITY

Is the Market ERP the correct premium for company-specific volatility?

$$TCOE = r_f + T\beta(r_m - r_f) = r_f + (\sigma_i/\sigma_m)(r_m - r_f)$$

Where $(r_m - r_f)$ is, of course, just the ERP [the return on the market (r_m) less the risk-free return (r_f)] that is commonly employed in all CAPM applications.

The CAPM formula of $k_e = r_f + \beta(ERP)$ makes use of the ERP because this is the appropriate premium a fully diversified investor would expect to earn from the public markets.[8] By definition, the ERP is that premium which is sufficient to induce an investor to put his or her capital into the public equity markets rather than into risk-free government bonds. Recall that the *ex post* ERP is calculated from a time-series history of actual market-wide returns (or, usually, the S&P 500 index returns as a benchmark for the preferable, but practically immeasurable, universe of all assets). As such, virtually all the company-specific volatility has been diversified out of the *ex post* ERP—the returns of the S&P 500 represent only systematic, market-wide volatility. Ergo, the ERP is a reward rate appropriate to systematic risk.

IS ERP APPROPRIATE?

But is the ERP the appropriate return to adequately compensate the notional investor to incur non-systematic risk?[9]

Consider, for example, an undiversified investor attempting to decide between two different publicly traded firms. Company A has historically been highly correlated to the movements of the S&P 500 (i.e., ρ^2, also called r-squared or the coefficient of determination, is widely accepted as representing the *ex post* proportionate amount of the proxy stock returns that are attributable to systematic risk. It ranges from 0.0 to 1.0, where 0.0 would indicate that none of that stock's price movements were related to systematic risk; 0.5 would indicate half were systematic and half company-specific, and 1.0 would indicate all were related to systematic, or market-wide, risks). Company A's ρ^2 is very close to 1.0 and has consistently shown a stock price volatility (σ_A) of 30 percent. Company B has also reported a steady volatility of 30 percent, but in this case its ρ^2 has been very low, virtually 0.[10] So even though both firms show the same relative degree of overall historic stock price dispersion, the fundamental difference between the two is that Company A bears virtually no idiosyncratic risk whereas Company B returns are driven almost entirely by company-specific risk.

Would investors demand the same rate of return for both firms? Supporters of the TCOE methodology would answer in the affirmative on the principal of economic substitution. In the case of Company A, this investor knows he cannot expect to earn more than the ERP rate, multiplied by the appropriate β, for only bearing systematic risk. And, if volatility is a good measure of overall investment risk, then both firms have historically incurred the same degree of total volatility. So if Company B paid a higher premium per unit of risk than Company A, savvy undiversified investors would quickly identify an opportunity for arbitrage profits and bid up the price of Company B until both firms were paying the same risk premium per unit of risk—namely, the ERP. This perspective seems reasonable under the assumption that a unit of systematic volatility is just as equally risky as a unit of company-specific volatility. But are they?

Note that the second term in the Market Beta CAPM formula is β(ERP), which

5 For example, BP's NYSE ADS security closed at $60.29 on April 20, 2010, the day its deepwater rig Horizon sank and began spewing millions of barrels of oil into the Gulf of Mexico (a very company-specific risk). Within 34 trading days that security had dropped 40 percent in value. The S&P500, conversely, had only lost 12.5 percent of its value over that time. The 60-month β of BP was 0.7 at the end of March 2010, suggesting that a 12.5 percent market decline should only foster an 8.75 percent systematic decrease in BP price.

6 William F. Sharpe, et al, *Investments: Second Canadian Edition*, Scarborough, Prentice Hall Canada, p. 486.

7 Ibid, p. 489.

8 Most precisely, in an *ex ante* application, the investor would require the forward looking expected rate of ERP $[E(r_m) - E(r_f)]$. Frequently, however, the very long-term historical (*ex-post*) actual $(r_m - r_f)$ is employed as a proxy for the unknown expected premium. Considerable debate can be levied against whether a long-term historical average is sufficiently predictive of future expectations—or, conversely, whether predicting future macroeconomic outcomes doesn't entail a vast degree of subjective reasoning. The expected vs. historical average issue is important but ancillary to this article.

9 Kasper approaches this same issue from a different perspective. He does not question whether the ERP is an appropriate rate of reward for idiosyncratic risk, but rather shows that Tβ, as proposed in the TCOE, falls on the Security Market Line, as does β; and then he insightfully asks, "How can the systematic risk of a dissimilar company also equal the total risk for the subject?" "Fallacies of the Bulter-Pinkerton Model and the Diversification Argument," *The Value Examiner*, January/February 2010, p. 18.

10 Note that only an undiversified investor would expect to be compensated for idiosyncratic volatility as he actually bears this risk. Theoretically, a fully diversified investor would prefer to purchase the risk-free asset rather than a perfectly uncorrelated equity, as the expected return from both would be the same. He would not purchase Company B.

can also be written $\rho(\sigma_i/\sigma_m)(r_m - r_f)$ and this, in turn, can be rearranged to:

$$\rho\sigma_i((r_m - r_f) \div \sigma_m)$$

Similarly, the second term of the TCOE formula is $(\sigma_i/\sigma_m)(r_m - r_f)$ and this can be rearranged to equal:

$$\sigma_i((r_m - r_f) \div \sigma_m)$$

Common to both terms is the multiplier $((r_m - r_f) \div \sigma_m)$ which should be quickly recognizable as the Sharpe Ratio. This is the market price of risk, or, when derived from ex post data, it was the excess returns the market produced per unit of market volatility. Since the $(r_m - r_f)$ numerator represents the ex post ERP, this reward rate is specific to systematic returns only.

In the case of Company A, where the $\rho^2 \approx 1$ (and therefore, of course ρ is also ≈ 1), the results of the TCOE and CAPM formulas will be exactly the same because:[11]

$$\sigma_i((r_m - r_f) \div \sigma_m) = 1\sigma_i((r_m - r_f) \div \sigma_m)$$

However, in the case of Company B, where $\rho^2 \approx 0$ (and ρ is also ≈ 0):

$$\sigma_i((r_m - r_f) \div \sigma_m) \neq 0\sigma_i((r_m - r_f) \div \sigma_m)$$

In fact, the right side of this inequality (the CAPM derived side) would be equal to zero. CAPM theory only speaks to what the appropriate expected rate of return should be for that portion of a security's excess return that originates from systematic volatility. When, as in the case of Company B, the expected systematic risk of a security is zero (i.e., that equity is perfectly uncorrelated with the movements of the market), then CAPM correctly predicts the appropriate expected systematic returns to that equity is zero.

CAPM theory provides no guidance whatever as to what the appropriate rate of return the non-systematic volatility of any asset should be rewarded with. The TCOE supporters have simply assumed that the systematic return per unit of risk (i.e., the *ex post* Sharpe Ratio) is also an appropriate rate to compensate non-systematic risk exposure. But they have done so on very little evidence. "There is surprisingly little empirical research on volatility at the level of the industry or firm."[12] And, as yet, the academic attention towards this issue is sparse and often contradictory.[13]

Hypothetically, if one were to *assume* that company-specific (idiosyncratic) risk is generally riskier than market-wide (systematic) risk, then the logical conclusion would be that the ERP is not sufficiently large enough to attract investors to bear non-systematic risk. The reason why this is so is that one can easily simulate any desired level of Total Beta risk and earn the ERP without ever being exposed to company-specific risks. For example, assume that a given investor is considering a 100 percent private equity position in a firm that has an expected (total) volatility of 30 percent. Moreover, assume the following market conditions:

r_f = 4.0%
ERP = 6.0%
σ_i = 30.0% (or a public market proxy for the expected σ of the private firm)
σ_m = 20.0%

TCOE = 4.0%+(30%÷20%)(6.0%)
= 13.0%

According to the BPM the Total Beta is 1.5 (30% ÷ 20%) and the resultant TCOE is 13.0 percent. However, this investor can easily simulate the expected outcome of this private equity investment simply by investing in the Market Index[14] with a combination of 100 percent of the funds initially planned for the private equity purchase plus an additional 50 percent amount of borrowed funds (assuming that one can borrow at the risk-free rate, which is consistent with CAPM theory). Now the investor's $E(r_p)$ is 13.0 percent and $E(\sigma_p)$ is 30.0 percent, precisely as it would have been had he purchased the private firm (see sidebar, "Expected Returns of the Market Index with Risk-Free Borrowing," page 19). The levered position in the Market Index virtually eliminates all exposure to company specific risks (even *ex post*[15]). The investor

11 In fact, the TCOE outcome always represents the CAPM results under the assumption that the subject equity is perfectly positively correlated with the Market Index.
12 Campbell et al, "Have Individual Stocks Become More Volatile?" Harvard University, 2000.
13 In Chen, et al, "Entrepreneurial Finance and Non-Diversifiable Risk," 2009 (NBER #w14848), the researchers' expectations were that "The entrepreneur demands an extra premium for bearing the idiosyncratic risks of the firm."

14 While there would have been a time when such a suggestion as "invest in the Market Index" would have been purely theoretical, the plethora of ETFs and hedge funds that track the various indexes now make such investments easy and transactionally cheap.
15 In his 1976 paper, "How Diversification Reduces Risk: Some Empirical Evidence" (Working Paper No. 28-73), R. Westerfield reports how daily-returns data from 489 common stocks show sequentially declining standard deviations as random portfolios of 1, 5, 10, 15, 20 and then 489 stocks are observed. "For portfolios including at least 15 different securities approximately 75% of the total possible reduction in standard deviation has taken place." (pg. 8)

has climbed up the capital market line and is in at least as good a position as had he purchased the private firm. He has effectively converted all the potential company-specific risk he would have been exposed to with the private firm into systematic risk. And he is still earning the ERP as a unit price of risk, the same reward rate that the TCOE would have offered him for purchasing the private equity firm.[16]

Under these economic assumptions, a review of the Company A, Company B, and Levered Position in Market Index all produce the same TCOE results:

COMPANY A

TCOE = $r_{f} + \sigma_A(ERP/\ \sigma_m)$ = 4% + 30%(6% / 20%) = 13%

COMPANY B

TCOE = $r_{f} + \sigma_B(ERP/\ \sigma_m)$ = 4% + 30%(6% / 20%) = 13%

LEVERED MARKET INDEX

TCOE = $r_{f} + \sigma_p(ERP/\ \sigma_m)$ = 4% + 30%(6% / 20%) = 13%

In each case, the rate of reward is the ERP-based market price of risk: $(ERP/\ \sigma_m)$. Under the presumptions of TCOE, the overall exposure to risk in each case is the same: $\sigma_A = \sigma_B = \sigma_p$ = 30%. Therefore it is accepted that each would be rewarded at the same rate. Is this assumption reasonable? Even though the historic rates of volatility are the same in all three cases, the source or "type" of Company B's volatility is completely different from the other two alternatives. Would a normally risk-adverse investor presume, just because the recent history of *ex post* volatility in Company B was the same as the position in the Levered Market Index, that the future risks associated with each would be identical? Would he presume that the probability of Company B becoming insolvent and worthless was just as likely as every firm in the S&P 500 simultaneously failing? Would he be indifferent either to placing 100 percent of his life savings either in Company B or in a levered position in the Market Index and being rewarded at the same 13 percent rate in either case?[17] Common sense would suggest: no.

If the comparison we were making were not Company B vs. the Levered Market Index, but rather, Company B, a \$200 million firm where σ_B = 30%, and Company D, a \$20 billion firm where σ_D = 30%, would we expect the market to demand the same cost of equity from both B and D? TCOE pronouncements would have us believe that the TCOE of both firms would be equal because their volatility standard deviations are identical.[18] However, it is much more likely that the market would demand a significant size premium for Company B—and is it possible that a component of this premium relates to how the market perceives and prices idiosyncratic risk exposure differently in small firms vs. large?

TCOE assumes that investors will demand the same rate of reward per unit of risk for both systematic and company-specific volatility. But if this were literally the case, what would ever induce the rational investor to take a position in any security other than the Market Index—much less a 100 percent equity position in a private entity?

CAN HISTORY PREDICT FUTURE RISK?

Is a historical trend in total company risk predictive of future risks? A key assumption, in order for Total Beta to be useful in business valuation, is that the historic ratio of σ_i/σ_m is indicative of future expectations. But specifically considering those equities where the largest proportionate amount of risk is non-systematic, is it reasonable that past stock price volatility will be subsequently incurred in a constant proportion with market volatility? Company-specific risk is, well, specific to each company. It is incurred as the result of events and occurrences that are particular to that individual firm and, by definition, has very little, if any, correlation with the movement of the market.

If a guideline firm has consistently shown a total annual volatility of 30 percent over the past five years compared with the market's 20 percent, is that a good indication that this relationship will continue into the future? Perhaps. Leaving aside all the statistical rationalizations, the argument against Total Beta is that it can be nothing more than the coincidental ratio between two basically unrelated variables σ_i and σ_m. Total Beta is, in effect, conveying the firm's total-risk-per-unit-of-market-

16 Mathematical proof: Investing in the Market Index gives an expected return of $r_f + \beta(ERP) = r_f + \rho(\sigma_p / \sigma_m)(r_m - r_f)$ and since $\rho = 1$ for the Market Index, then the expected return is $r_f + (\sigma_i / \sigma_m)(r_m - r_f)$, which is, of course, the TCOE formula.

17 For simplicity, I have not dealt with the value-at-risk difference in this comparison. That is, there are more absolute dollars at risk to this investor in the event of a complete failure of the S&P 500 than had he chosen Company B. Professor Damodaran suggests that the price of a market failure, or "catastrophic risk," is already included in the ERP. This would partially address the VaR issue.

18 "Total Beta is concerned with volatility of returns, so it captures total risk—all systematic risk, size risk, and company-specific risk." Quoted from "There is a New Beta in Town," by Butler & Pinkerton, *BV Update*, Vol.15, No. 3.

risk.[19] On an intuitive level, we need to ask why the primary cost of capital yardstick applied to a *private* equity transaction would be an all-encompassing measure of *public* market risk (namely σ_m). Unfortunately, the best answer may be "Because that's all we have." While we might far prefer to be able to assess the volatility of a proxy firm against the volatility of the entire private equity market, this denominator value is simply not observable.

To assess the predictive nature of Total Beta, I selected 26 firms at random,[20] and calculated their historical Total Beta

19 In his 1994 paper, "The Sharpe Ratio," Sharpe explains why the numerator in the ratio should always be a differential and not simply a uni-dimensional variable, as it is in the case of Total Beta. "While such a 'return information ratio' may be useful as a descriptive statistic, it lacks a number of the key properties of what might be termed a 'differential return information ratio' and may in some instances lead to wrong decisions."

20 Selected from the Russell Micro-cap Index: One for each letter of the alphabet provided that certain pre-determined liquidity ratios and trading history criteria were met. I reasoned that micro-cap firms would be the most likely proxies for private firms.

TABLE 1: 26 FIRMS RANDOMLY SELECTED FROM RUSSEL MICRO-CAP INDEX

			Developed from month-end data			
Symbol	**Firm Name**	**# Years of Data**	**Annualized Standard Deviation**	**Market Beta**	**R-Squared**	**Total Beta**
AAON	AAON INC	10.0	33.6%	0.82	0.15	2.08
BSDM	BSD MEDICAL CORP	10.0	63.4%	0.77	0.04	3.92
CAW	CCA INDS INC	10.5	41.1%	0.78	0.07	2.54
DDIC	DDI CORP	6.5	53.7%	0.76	0.05	3.25
EMCI	EMC INS GROUP INC	10.0	33.2%	0.78	0.14	2.09
FPIC	FPIC INSURANCE GROUP INC	10.0	48.7%	0.57	0.04	2.89
GPX	GP STRATEGIES CORP	10.9	40.7%	0.56	0.05	2.47
HQS	HQ SUSTAINABLE MARITIM	6.5	96.2%	0.55	0.01	5.36
IDT	IDT CORP	9.5	64.5%	1.57	0.15	4.04
JAX	J ALEXANDERS CORP	6.5	46.1%	1.37	0.22	2.94
KFED	K FED BANCORP	6.3	28.7%	0.47	0.07	1.73
LCAV	LCA-VISION INC	10.0	78.6%	2.35	0.23	4.86
MFRI	MFRI INC	10.0	53.8%	0.89	0.07	3.33
NCS	NCI BUILDING SYSTEMS INC	10.9	69.2%	1.68	0.15	4.28
CHUX	O CHARLEYS INC	10.0	63.3%	1.62	0.17	3.92
PTSI	P A M TRANSN SVCS INC	10.0	48.8%	1.26	0.17	3.02
QCRH	QCR HOLDINGS INC	8.7	31.4%	0.37	0.04	1.96
RAS	RAIT FINANCIAL TRUST	10.9	76.2%	1.88	0.20	4.26
SCBT	SCBT FINANCIAL CORP	10.0	27.6%	0.74	0.19	1.71
TESS	TESSCO TECHNOLOGIES INC	10.0	42.1%	0.98	0.14	2.60
UFPT	UFP TECHNOLOGIES INC	10.0	59.4%	1.48	0.16	3.67
VIST	VIST FINANCIAL CORP	10.0	26.9%	0.39	0.05	1.66
WCAA	WCA WASTE CORP	6.0	47.9%	0.70	0.06	2.96
XRIT	X-RITE INC	10.0	55.3%	0.49	0.02	3.35
YAVY	YADKIN VALLEY FINL CORP	10.0	42.3%	0.95	0.14	2.56
ZLC	ZALE CORP	10.9	68.3%	1.76	0.18	4.16
	AVERAGE		**51.6%**	**1.02**	**0.11**	**3.14**

TABLE 2: COMPARISON OF 60 MONTH ROLLING BETAS AND TOTAL BETAS

Symbol	Firm Name	60 Month Rolling Beta CV	60 Month Rolling Total Beta CV
AAON	AAON INC	0.168	0.184
BSDM	BSD MEDICAL CORP	1.047	0.198
CAW	CCA INDS INC	0.668	0.253
DDIC	DDI CORP	0.156	0.095
EMCI	EMC INS GROUP INC	0.213	0.205
FPIC	FPIC INSURANCE GROUP INC	0.516	0.399
GPX	GP STRATEGIES CORP	0.977	0.123
HQS	HQ SUSTAINABLE MARITIME	0.239	0.197
IDT	IDT CORP	0.441	0.318
JAX	J ALEXANDERS CORP	0.055	0.024
KFED	K FED BANCORP	0.068	0.030
LCAV	LCA-VISION INC	0.216	0.165
MFRI	MFRI INC	0.249	0.235
NCS	NCI BUILDING SYSTEMS INC	0.464	0.245
CHUX	O CHARLEYS INC	0.408	0.309
PTSI	P A M TRANSN SVCS INC	0.993	0.126
QCRH	QCR HOLDINGS INC	0.445	0.184
RAS	RAIT FINANCIAL TRUST	0.573	0.402
SCBT	SCBT FINANCIAL CORP	0.254	0.187
TESS	TESSCO TECHNOLOGIES INC	0.295	0.255
UFPT	UFP TECHNOLOGIES INC	0.620	0.206
VIST	VIST FINANCIAL CORP	0.468	0.149
WCAA	WCA WASTE CORP	0.074	0.012
XRIT	X-RITE INC	0.632	0.313
YAVY	YADKIN VALLEY FINL CORP	0.826	0.308
ZLC	ZALE CORP	0.662	0.469
	AVERAGE	**0.451**	**0.215**

Number of CV Observations < 0.25	9	9
Number of CV Observations < 0.5	16	26
Number of CV Observations < 0.75	22	26
Number of CV Observations < 1.0	25	26

and Market Beta, as shown in Table 1.

In addition, the coefficient of variation (CV) was determined for each of the 60-month rolling Betas. The goal was to determine which statistic demonstrated the highest proportionate degree of variability over time. The results of those observations are shown in Table 2.

In only one of the 26 cases was the Market Beta CV lower (AAON, and only marginally so) than Total Beta CV. Clearly, historic Market Beta, in our random sample, has exhibited a much wider range of variation over time than has Total Beta. Both Market and Total Beta reported nine of the 26 observations with CVs less than 0.25. However, all of the Total Beta CVs were less than 0.5, whereas only 61 percent (16 out of 26) of the Market Beta observations were less than 0.5. The generic formula for CV is σ_i divided by the sample mean—the lower the CV, the less the proportionate amount of dispersion in the historic Betas. And since the sample mean would also be the expected value of the rolling Beta statistic, we can also generally say that a one-standard-deviation range around the expected Market Beta would be the average Market Beta plus or minus 45 percent (since the overall average of Market Beta CVs is 0.4509). Conversely, for Total Beta, a one-standard-deviation range would be the expected Total Beta, plus or minus 21 percent. If we assume that the Market and Total Betas are normally distributed for this sample of micro-cap firms, we would then predict (via the empirical rule) that approximately 68 percent of all probable outcomes fall within one standard deviation of the expected mean.

Our informal random sample of 26 firms does very much support the Butler and Pinkerton contention that Total Beta is much more "stable" than are the Market Betas for the same firms. "The term that creates the most noise in the traditional Beta estimate is the estimate of correlation between the asset and the market index. The relative volatility measure [Total Beta] does not require a correlation measure and hence is less noisy."[21] It would appear that Total Beta does offer a higher inferential value than does Market Beta for the micro-cap asset class sample data tested. This may or may not be true for other asset classes. More empirical research in this area would be valuable to the valuation community.

BETA QUALITY

We should be grateful to Butler and Pinkerton for bringing the BPM and Total Beta to such a level of professional notoriety. In doing so, they have also highlighted a shortcoming of the traditional *ex post* Market Beta that many practitioners have ignored. According to Butler and Pinkerton, "Market Beta is too unstable to use reliably with any meaningful insight. Even when using the same historical look-back period and return interval frequencies, Market Betas often show extreme volatility."[22] As an example, they show NASDAQ stock DYII that reports a β that ranges between -0.47 to 1.18, depending on which day of the week the sample data are regressed to. During this same period, DYII's Tβ ranged between 10.72 and 12.93, according to Butler and Pinkerton.

Volatility or inconsistency in sample-to-sample comparisons is a shortcoming of *ex post* Market Betas. This problem does not invalidate the use of *ex post* measurements as a prediction of *ex ante* outcomes, but it does put the onus of responsibility on the appraiser to know the quality of his or her Beta estimate. Market Betas that demonstrate a high degree of variability over time should be deemed unreliable and disqualified from use as a proxy.

One possible explanation for weekday Beta variability may be that regressions that employ weekly measurement frequency (as Butler and Pinkerton have employed in their writings), rather than monthly, are inherently more noisy. I tested this theory by calculating 60-month Betas for the 26 sample firms and comparing the results against the 260-week Betas using the same time period and same weekday-end-of-sample point. While the monthly measurement frequency narrowed the range of dispersion compared to the 260 week observations, in both cases (260 weeks and 60 months) the Market Beta still showed a considerably wider range than did Total Beta.

Historical Betas are useful when they are believed to predict future relationships. Therefore, the problem of Beta instability is not limited just to weekday-to-weekday comparisons. One would also wish relative stability over a range of sample look-back periods as well—that is, if one were regressing month-end data over a 60-month sample period and obtained the results shown in Table 3, which uses three overlapping look-back periods.

We should be highly suspicious about the reliability of the Stock X regressions.

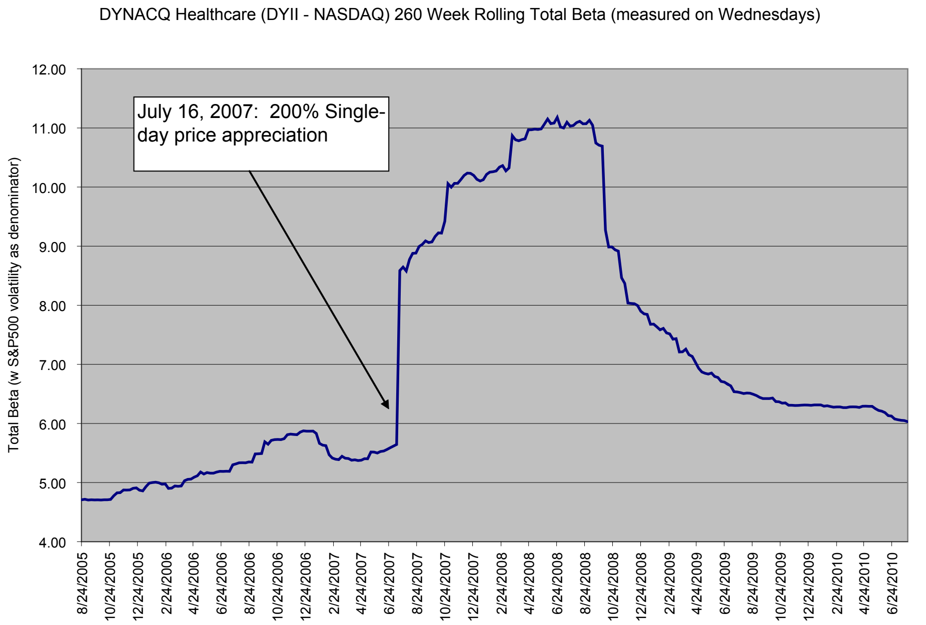
21 A. Damodaran (n.d.), "Estimating Risk Parameters," retrieved from New York University, Stern School of Business website: http://pages.stern.nyu.edu/~adamodar/New_Home_Page/papers.html.

22 Peter J. Butler and Keith A. Pinkerton (n.d.), "Total Beta: The Missing Piece of the Cost of Capital Puzzle." Retrieved from Hooper Cornell Valuation website: http://hoopercornellvaluation.com/FileUpload/Butler_Pinkerton_Checkpoint.pdf.

TABLE 3: HYPOTHETICAL *EX POST* STOCK REGRESSION

60 Month Beta - Sample Period Ending	Stock x	Stock y
30-Jun-06	1.3	1.6
30-Sep-06	0.8	1.5
31-Dec-06	1.1	1.4

GRAPH 1: DYII ROLLING TOTAL BETA HISTORY, UNALTERED



Such a substantial swing in the slope of this fit line, in response to a minor change of six months' worth of data,[23] should at least cause us to question the reliability of employing Stock X as a predictor of future results. (The presumption here is that one has considered any external factors that might have reasonably accounted for such a dramatic shift—such as a wholesale change in the capital structure of the firm, or major acquisition/divestiture, or substantial macroeconomic shift in the market index itself). In contrast, the directional change in the Stock Y regressions is either (a) the result of the fact that stock Betas do change over time, or (b) is caused by non-systematic price distortions in the sample data.[24] In either case, we would be less concerned with the reliability of Stock Y's regressions.

The issue of Beta quality is not exclusively limited to β. To be predictive, *ex post* Tβ also needs to display a relative degree of stability over time. For example, we can visually examine DYII *ex post* Total Beta, (shown on a rolling 260 week historical basis) in Graph 1.

Would we instinctively conclude that future Total Beta would be expected to range between 11 and 13 as Butler-Pinkerton have measured it?[25] The average Tβ throughout the sample is 7.1. In fact, the results obtained by the BPM were largely influenced by a single outlier that occurred July 17, 2007 (where single-day price appreciation was in excess of 200 percent). If this single outlier is artificially eliminated from the dataset (e.g., the net return for that week is set to zero), a substantially different Total Beta profile is achieved, as shown in Graph 2. With the removal of a single week of DYII volatility, the average rolling Tβ throughout the sample is 5.32.

Faced with such a historical profile, which Tβ would the appraiser select to represent future expectations: 5.3, 7.1, or 12.0? Or would the conclusion be lower than any of these observations based on the premise that the most current data are trending downward? The point here is that Tβ provides no panacea of certainty—it is going to be subject to many, if not all, of the foibles and shortcomings of other market measurement techniques and often for exactly the same reasons.

On the issue of the predictive quality of Total Beta, there were instances in the 26-firm micro-cap sample where Tβ volatility had been reasonably stable throughout the 10-year firm history, and arriving at a prediction of expected future Tβ would be relatively painless. Just as frequently, however, the standard deviation of the Tβ history would show a significant level of dispersion, even over a relatively short period, and an appraiser would, as in the case of DYII, need to exercise considerable professional judgment in the prediction of future Tβ. All of the Tβ profiles showed a moderate to large increase in Tβ ratio in the July 2007 through January 2009 period, leaving the appraiser to wonder if and how much his estimate of Tβ should incorporate the possibility of another future global recession. An example of the BSDM rolling 60 month profile is shown in Graph 3.

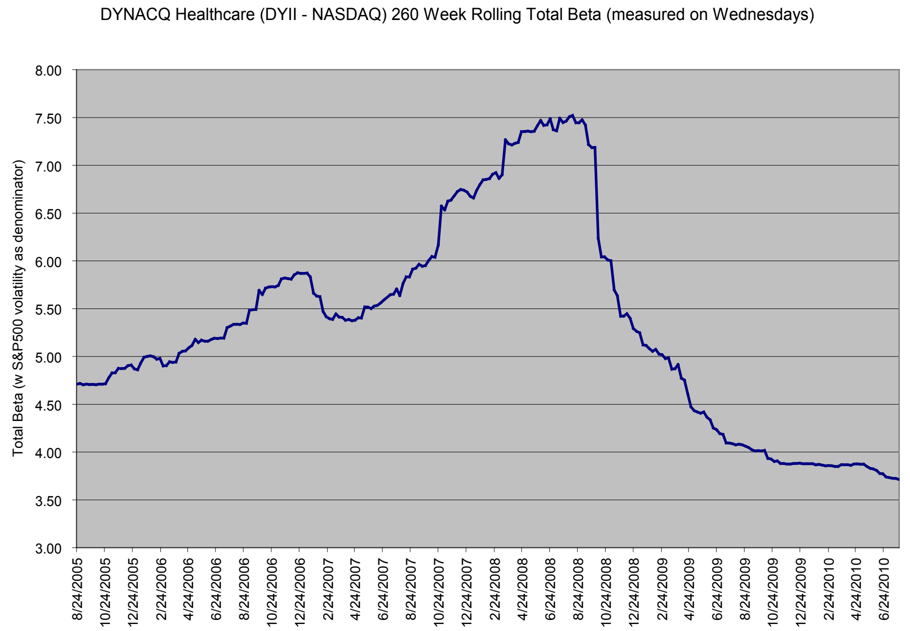
Until June 2008, the rolling ρ^2 of BSDM is virtually zero. Therefore almost none of the BSDM stock price re-

23 In the transition from Stock X sample one to sample two, for example, the months of July through Sept. 2006 are being substituted for July through Sept. 2001. Such a minor change in source data would normally be expected to have only a minimal impact on *ex post* Beta.

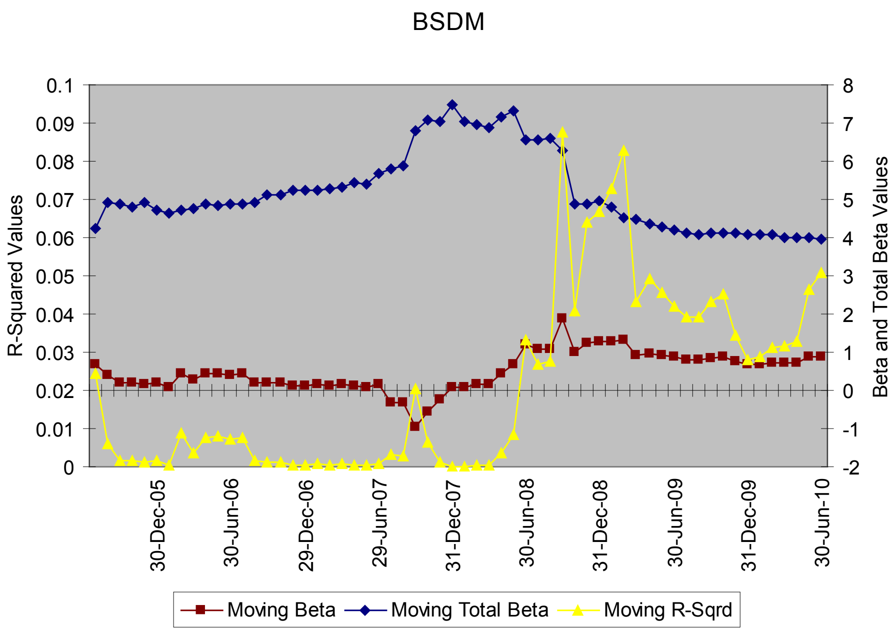
24 For a more detailed discussion on the topic of non-systematic noise in Market Beta regressions, see Richard R. Conn, "Sensitivity of a Stock Beta to Non-Systematic Price Impacts," 2006, at www.accessioncap.com/caseStudies/Stock_Beta_Sensitivity_To_Nonsystematic.pdf.

25 For securities with high levels of volatility (e.g. the annualized σ of DYII, using a 10 year history, was over 100 percent per year), a marked difference in the Total Beta profile will result if the natural logarithm of the week-over-week delta is employed compared with when the arithmetic proportional change is used (the graphs above use this latter method in order to be consistent with the Butler-Pinkerton approach). Moreover, if a monthly rather than weekly frequency had been employed, the profile would be greatly smoothed and the extremes softened.

GRAPH 2: DYII ROLLING TOTAL BETA HISTORY, WITH OUTLIER ELIMINATED



GRAPH 3: BSDM ROLLING HISTORY OF BETA, TOTAL BETA, AND R^2



turns are attributable to systematic risk. Logically, then, why would anyone rely on the β statistic, a measure of systematic market risk, in such a case when the vast majority of the risk exposure stemmed from non-systematic causes? In these specific instances, Tβ will be a more meaningful risk metric, and perhaps an even more useful measure will be σ_{BSDM} alone.

It is entirely appropriate that Butler and Pinkerton have brought the issue of Beta quality to center stage. The practice of referencing a Beta book or a Bloomberg report and unquestioningly accepting that single point-in-time representation of Market Beta as "the truth" is perilous. If the Butler Pinkerton Calculator serves no other purpose than to allow practitioners to efficiently check the historical stability of a guideline firm's Market Beta, then on this count alone it should be a very useful tool to the profession at large.

A DEFENSE OF 'TRUE' BETA

Some may argue that CAPM-derived Beta suffers from the same weakness as Total Beta in that it relies on examining the relationship of market index-to-stock returns of the past and inferring that this can predict the future. This is true. And it is widely accepted that firm Betas do change over time, so the observations of the past will never provide perfect fidelity of actual performance in the future. However, there is a subtle but important difference between Beta and Total Beta. Beta attempts to uncover a causal relationship between movements in the market and those of the firm under observation. A "true" Beta (i.e., one that is free from the measurement distortions caused by idiosyncratic risk and other errors) of 1.5 is inferring that the excess returns of, say, +2.0 percent in the market will be reflected as a +3.0 percent excess return in the subject firm. As Butler has often pointed out, such a predictive quality in Beta is in no way useful in determining what the *company-specific* risks may be in that particular firm. However, if a reasonable degree of reliance can be placed on the predictive nature of Market Beta (and here we must be speaking of the imperfect *ex post* Market Beta, as there is no means of actually observing "true" Beta), then this knowledge will be indispensible to the business appraiser in assessing that firm's systematic cost of capital.

Through the statistical process of linear regression, the analysis of subject stock returns are compared with same-period market index returns to determine the strength of the linear relationship between the two. The concept is that if there is a well defined and consistent proportional relation between how the market moved, compared with how the subject firm's stock price responded, then the *inference* is that the former may be the determinate of the latter—that there is a causal relationship between market returns and the firm's stock returns.

In contrast, for Total Beta there can not even be an inference of causality between the overall volatility of the Market (σ_m) and the company-specific portion of σ_i. The two are simply independent.

CONCLUSIONS

In spite of quite a small sample size, selected randomly but without much rigor, the observations of the 26 micro-cap firms definitely support Butler and Pinkerton's contention that *ex post* Total Beta is much less volatile than are the Market Betas measured for the same firms over the same periods.

Contrary to much of the previous writings that have focused upon debunking the veracity of Total Beta, the findings of this paper have questioned the theoretical validity of TCOE. The TCOE formula rests on the assumption that investors would demand exactly the same return-per-unit-of-volatility (ERP/ σ_m) for un-diversifiable, company-specific risk as they do for systematic risk. As yet, there is very little empirical evidence to support this position. At the extremes, when comparing a Market Index position to a small micro-cap entity whose *ex post* volatility has primarily been idiosyncratic, it seems implausible that any rational investor would demand the same return on equity for both, simply because their historic volatility rates have been equal (or have been engineered to be equal). Logically it would seem that private equity investors would demand higher reward premiums for their exposure to company-specific risk than just the ERP.

At least we can say with certainty that, on a per-unit-of-risk basis, public company investors will also be compensated at the ERP rate, and will always enjoy much greater flexibility to choose exactly how much un-diversified risk exposure they care to assume, if any. So what would possibly motivate them to invest in private equity if the expected per-unit risk rewards are no higher? The difficulty of objectively quantifying idiosyncratic risk has been with us a long time. As yet, however, we don't seem to have a good enough understanding of how the public markets price this highly complex risk exposure, much less the private markets. Until we do gain this higher understanding, the TCOE employment of an ERP-based rate of return for both types of risk appears to be founded more on convenience than on any defensible economic reasoning.

Total Beta does appear to be more stable than Market Beta. And even if Total Beta does not statistically meet the technical definition of a "Beta" (i.e., a coefficient used to lever a risk premium), it does appear to convey useful information about the overall *ex post* risk profile of the proxy firm under investigation. So, why wouldn't a diligent and impartial business appraiser employ Total Beta as just one more tool to consider in the measurement of overall firm risk? If the simple ratio of firm volatility over market volatility provides us with a greater insight into what has happened to a proxy firm in the past, and perhaps allows us to make reasonable inferences as to what risks it may face in the future, why ignore it? Total Beta may not be, as Butler has contended, "the sharpest tool in the appraiser's toolbox." Yet, with so few instruments to rely on, the profession cannot afford to dismiss any means that might offer another insightful perspective upon quantifying risk.

Most importantly, a spotlight on the issue of Market Beta quality should cause us to be much more critical over the acceptance of market statistics. A regression may be run from the covariance of any two data sets, regardless of how remote and completely unrelated the data might be. It does not necessarily follow that the resultant slope of the fit line is actually conveying useful information. Those results may have occurred purely through random chance. A different look-back period or different measurement frequency might produce an entirely contradictory outcome.

Similarly, Total Beta is also subject to measurement "quality" issues. If the results are unreasonably skewed by one data outlier, or if slightly different look-back periods or measurement frequencies produce a widely ranging statistic, then one needs to question whether there is any inferential value in these data at all. In the worst-case scenario, where the period-to-period comparison of Total Beta is highly erratic, the best conclusion may be that this firm is subject to wide swings in volatility over short periods. And this information, of its own accord, is useful in coming to some kind of conclusion over the relative riskiness of the firm. VE

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Sidebar

EXPECTED RETURNS OF THE MARKET INDEX WITH RISK-FREE BORROWING

By Richard R. Conn, CMA, MBA, CPA/ABV

A two-security portfolio comprised of the Market Index and the Risk-Free Bonds can be designed to simulate the returns expected from any private equity firm paying a per-unit risk rate of the equity risk premium (ERP). More importantly, this portfolio can duplicate these private company expected returns and volatility and still insulate the investor from virtually *any* exposure to non-systematic risk. We are attempting to simulate the returns of a private firm that has an expected TCOE of 13.0 percent and total volatility of 30 percent, as represented by some facsimile public proxy company, where:

ERP = 6.0%
r_f = 4.0%
σ_m = 20.0%
σ_i = 30.0%

Therefore:

TCOE = 4.0% + (30.0 / 20.0)(6.0%) = 13.0%

EXPECTED RETURNS

Recall that the expected returns on any two security portfolio is simply a weighted average of the two *E(R).* And, if X_m represents the percentage of the portfolio invested in the Market Index, and $(1 - X_m)$ represents the percentage invested in risk-free bonds (and if this percentage is negative, that indicates borrowing rather than lending at the risk-free bond rate), then:

$$\begin{aligned} E(R_p) &= X_m (r_f + \beta(ERP)) + (1 - X_m)(r_f) \\ &= X_m (4.0\% + (1.0)(6.0\%)) + (1 - X_m)(4.0\%) \\ &= 150\%(10\%) + (1 - 150\%)(4.0\%) \\ &= 13.0\% \end{aligned}$$

EXPECTED VOLATILITY

The standard deviation for a two-security portfolio is:

$$\sigma_p = \sqrt{[X_m^2 \sigma_m^2 + (1 - X_m)^2 \sigma_{rf}^2 + 2 X_m (1 - X_m) \rho_{mrf} \sigma_m \sigma_{rf}]}$$

However, because the variance on the risk-free borrowing is always zero (i.e., there is never any doubt as to how much debt will be owing at any given time, it will always be the principal borrowed plus the accrued interest at the risk-free rate), then $\sigma_{rf}^2 = \sigma_{rf} = 0$, and as a result the above simplifies to:

$$\sigma_p = \sqrt{[X_m^2 \sigma_m^2 + 0 + 0]} = \sqrt{150\%^2 (20.0\%)^2} = 30.0\%$$

Ergo, by investing 150 percent of one's disposable funds into the Market Index (comprised of 100 percent of the investor's own funds, plus another 50 percent borrowed at the risk-free rate), an investor may simulate the expected return (13.0 percent) and volatility (30.0 percent) of a private firm. However, virtually all of the company-specific risk has been diversified out of the Market Index. Therefore, by investing in the simulated portfolio the investor has, for all practical purposes, completely insulated himself against company-specific risk.