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SENSITIVITY OF A STOCK BETA TO NON-SYSTEMATIC PRICE IMPACTS:
And their implications in the use of the CAPM method of determining Cost of Capital

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EXECUTIVE SUMMARY:

- The purpose of the paper is to consider how non-systematic or company specific stock price movements (noise) distorts regressed Beta (β)
- There is no reliable method of separating systematic vs. non-systematic price impacts from historical stock data
- Accurate application of the CAP-Model requires attaining an estimate of β that is free from non-systematic distortion
- Conclusion i) Even small levels of random noise added to otherwise 'pure' share price data causes significant variation in the regressed β
- Conclusion ii) β derived from single share regressions are probably unreliable
- Conclusion iii) A more accurate prediction of β can be derived by finding a collection of comparable public securities in the same industry with similar risk structures and calculating a weighted-average of the individually regressed betas

INTRODUCTION

When looking at a beam of white light it is impossible for the naked eye to discern the presence of the primary colours, much less distinguish the relative proportions of each of those colours.

Similarly, when attempting to ascertain the Beta (β) of a publicly traded security[1] by regressing historical stock prices against a broadly-based market index, the results obtained mathematically will be a combination of a number of different elements. In the light ray analogy, it would be possible, of course, to direct the ray through a prism by which the existence of the primary colours then become discernable to the human eye. Further, it would be possible to conduct more sophisticated spectrographic analysis that could very precisely determine the relative proportion of all the colour elements in that particular beam of light.

In the case of a β derived through means of a linear regression, however, there is no convenient prism to reveal the presence of the two primary elements of stock-price movement: systematic and non-systematic risk (systematic risk relates to the movement of entire market, and is sometimes referred to as non-diversifiable risk, whereas non-systematic is the company-specific risk and is considered to be diversifiable).

CAPM – An Objective means of Estimating a firm's Cost of Capital

The reader is assumed to have a fundamental understanding and acceptance of the Capital Asset Pricing Model (CAPM) and all its basic tenets. The most fundamental premise of the CAPM is that the *expected* return on individual equity investments should change in some relative proportion with the Market as a

[1] In this paper we will concern ourselves solely with the estimation of the Beta for publicly traded equities. However the theory could be applied to the Beta of any given asset, real or financial.

whole. Like all models, CAPM is a simplification of reality, but it can be argued that this is the true purpose of any model. CAPM takes the multitude of variables that could impact the movement of a security price relative to a movement in the market portfolio and distills that down to an easily comprehended function of only two terms (three terms, if the error term is considered, as will be discussed later).

Recall that the CAPM formula is:

$$k_e = R_f + \beta \text{ x } (MRP)$$

Where:

k_e	= the EXPECTED return on the equity[2]
R_f	= the appropriate Risk-free rate of return
β	= that security's specific Beta
MRP	= the Market Risk Premium (i.e. the return the market is producing in excess of the risk-free rate)

<u>BETA</u>

The β of the Market as a whole is 1, of course, and any equity that has a β of 1 would, in theory, be expected to mirror the returns earned by the Market. So if the Market ascended 5.0% in any given month the β1 equity price would also be expected to climb 5.0% in the same period. However, equities with a published β of 1 (or approximately 1) seldom tend to perform this way in reality. This is because individual stock prices move both in relation to broad-based economic market conditions (i.e. the systematic drivers), as well as in relation to the company-specific impacts (i.e. the non-systematic drivers).

The CAPM model makes no allowance for k_e to include any provision for company-specific/non-systematic risks. In fact, the theory presumes the investor will be able to diversify away ALL non-systematic risk by holding an optimal weighting[3] of all the equities in the Market Portfolio. Because the risk of individual company/non-systematic risks can be effectively eliminated through broad investment diversification, the

[2] It should be noted that, if the Beta has been regressed from the data of an equity that holds a balance of both Debt and Equity capital, the Return on Equity derived will be 'Levered' (i.e. risked for the presence of debt), and therefore the Beta regressed will also reflect the existence of debt.. Conversely, if the capital structure of the firm was 100% equity throughout the period of the regression, the Beta and derived Return on Equity will be Un-Levered. This added refinement is not germane to our primary concern here, however.

[3] The investor is presumed to be able to purchase the Market Portfolio where each security is held in a proportion that produces a risk-return ratio that dominates all other possible combinations. Having identified this optimal Market Portfolio investment, the individual investor would tailor to his own risk/reward tolerances by either borrowing or lending at the risk-free rate. A less risk-adverse investor would borrow at the risk-free rate and then buy more of the Market Portfolio with the borrowed funds – thereby increasing his/her leverage and potential for Market gains and losses. A more risk-adverse investor would lend to the risk-free market, preferring the certainly of the virtually guaranteed returns and putting a much smaller share (if any) of his/her wealth at risk in the Market Portfolio.

CAP-Model postulates that the Market will not reward the investor for taking any company-specific risk. So, if there is no provision for non-systematic risk in k_e , it is a mathematical certainty that there can be no company-specific risk element built into the β x (MRP) part of the equation.

Just because CAPM presumes the widely-diversified investor should not be compensated for company-specific risk does not mean that individual stocks are not dramatically impacted by this type of risk. We know that when a bio-technology firm, for example, is granted FDA approval on its new health product, it's share price generally incurs a instantaneous and substantial non-systematic appreciation. Conversely, should be same firm be named in a credible class-action product liability suit, it's share price generally incurs a substantial decline. So the problem becomes, if actual historical stock prices reflect an unknown blend of both systematic and non-systematic price impacts, how can the distortion of the latter be eliminated such that a 'true' β - reflective only of systematic risk, be regressed from the data? Unfortunately, the answer to this question is that there probably is no reliable means of stripping the company-specific or non-systematic price changes out of historical stock data[4].

PURPOSE OF THIS INVESTIGATION

What can be considered, and the subject of the rest of this paper is:

i) How sensitive is an analytically derived stock β to the distortions of non-systematic price fluctuations (and other 'noise' elements)?

ii) Could the regression of any actual single stock price history be expected to yield a dependable estimate of the 'true' β?

iii) Are there any analytical techniques that can be applied in the data collection or processing phases that tend to improve the reliability of the derived β?

METHOD: CREATE A HYPOTHECTICAL STOCK PRICE HISTORY

The methodology employed in addressing each of these questions will start by creating a hypothetically 'noise-free' data set. We will use the S&P/TSE[5] market index as a proxy for the Market Portfolio, and from this devise 60 months[6] of historical stock prices that are completely free of any non-systematic price

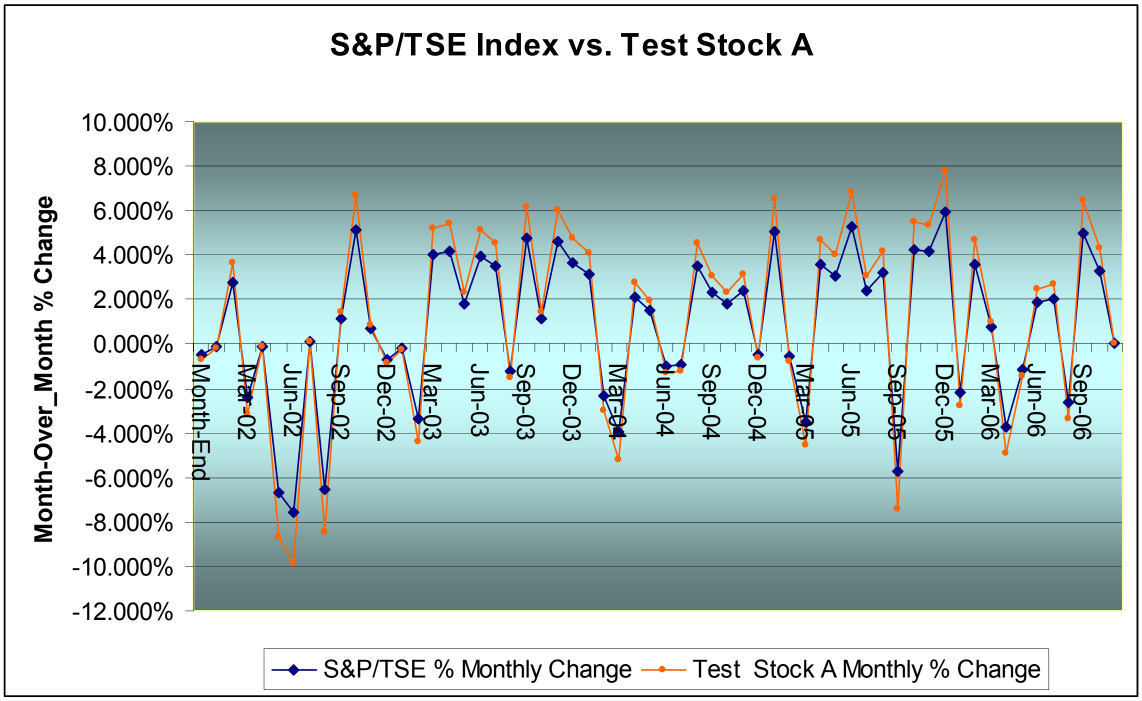
[4] "It may seem logical to examine historical returns on securities in order to determine whether or not they have been priced in equilibrium, as suggested by CAPM. However, the issue of whether or not such testing of the CAPM can be done in a meaningful manner is controversial." from William Sharpe et. al. *Investments – 2nd Canadian Edition"* pg. 244

[5] There is considerable controversy as to whether such a narrow representative of the entire Market does not introduce even more distortion into the process. However, the S&P/TSE Composite Index is the broadest based index existing in Canada that is used as a benchmark for almost all aspects of the Canadian economy. Based upon its general acceptance, then, we will continue to use this index upon which to regress the hypothetical stock histories.

[6] Beta regressions of 60 monthly observations are widely used amongst the financial services that publish this information. A simplifying assumption of all these regressions are that the Beta under investigation remains invariant over the regression time period. Individual firm betas do, however, change significantly

impacts. In other words, an artificial stock price history will be created that exactly duplicates the relative changes in the actual monthly movements in the S&P/TSE Index. The period of study includes the month-end S&P/TSE Index data of January 2002 through December 2006. A hypothetical equity has been assumed to start a $25/share on December 31, 2001 and then parallel the S&P/TSE Index throughout the next sixty months. For convenience, the equity is presumed not to pay dividends – all returns are expected to be earned in the form of share price appreciation. The initial test case assumes this stock has a constant β of 1.3, therefore in any month that the S&P/TSE Index increases 1.0%, the hypothetical test stock will ascend 1.3% in price over the month. Conversely, if the S&P/TSE Index decreases 2.0% in a given month, the test stock will decline 2.6% over that same period. The symmetry in this relationship can easily be seen in the following graph:

GRAPH A – S&P/TSE Index History and perfect correlation with a 1.3 β Stock



The R-Squared[7] of a linear regression of Test Stock A in its current state to the S&P/TSE index is precisely 1.0. R-Squared as a statistical measure ranges from zero to one, and the higher the value, the more precisely the regression has been able to fit a line to the data. This means the fitted line resulting from the method-of-least-squares regression has landed precisely on each of the 60 data points in the original sample. There are no "residual errors" (i.e. differences between the sample, y_i observation and the corresponding y value on the fitted line) that the regression has not been able to include on the fitted line. Here, of course, we had constructed the Test Stock A to move in direct proportion with the S&P/TSE

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over time and this tends to add more distortion to the regression conclusions. See http://socrates.berkeley.edu/~craine/econ137_07/Webpage/Gomez%20paper.pdf

[7] R-Squared is also referred to as the Coefficient of Determination and greater detail to it's relevance in beta testing is given in the Accession paper "R-Squared and T-Statistics as Predictor of Beta Accuracy".

Index, so a 1.0 R-Squared had to result. It is important to realize that the slope of the fitted regression line represents the derived β value. Therefore, a very low R-Squared value would indicate that the regression has been unable to determine a strong linear trend in the data and the resultant β may not be reliable. It is also not surprising that a statistical regression with the S&P/TSE Index as the independent X variable and Test Stock A as the dependent Y variable returns a β of 1.3 precisely, as this value was used in the function to arrive at the Test Stock A sample data. We have constructed the perfect CAPM individual share price history – free of any distortions that might have been caused by non-systematic risk[8].

What would happen to these relationships if we now introduce a random error term in the Test Stock A data? Such an outcome is introduced in order to emulate the possibility of non-systematic influences on the stock price. This random error is allowed to shift the Test Stock A month-end stock price to any amount *up to* plus or minus 10%[9]. Such a shift in monthly expected stock price as a result of company specific impacts at a 10% maximum would not normally be considered improbable. Further, in order to make the comparison more meaningful, the same random simulation is run simultaneously 10 times (Test Stock A through J) and presented below:

TABLE A - SIMULATED 60 MONTH PRICE HISTORY: w +/- 10% Random Noise

Instance	Observed β	R-Squared	Effective Annual ROR
TEST A	0.964	0.188	5.6%
TEST B	1.344	0.337	5.4%
TEST C	1.248	0.299	6.4%
TEST D	1.613	0.400	29.7%
TEST E	1.290	0.282	-3.8%
TEST F	0.896	0.204	7.7%
TEST G	1.137	0.245	18.1%
TEST H	1.290	0.306	0.3%
TEST I	0.840	0.206	-8.9%
TEST J	1.273	0.331	3.0%
Simple Average	1.190	0.280	6.4%
Minimum Observed	0.840	0.188	-8.9%
Maximum Observed	1.613	0.400	29.7%
Range	0.773	0.212	38.6%

[8] This would be known as a deterministic model, as there is only one specific dependent Y value for every possible independent X value.

[9] β of any given security (a) will be equal to the covariance of that security relative to the Market Portfolio (MP) divided by the variance of the Market Portfolio: $\beta = \sigma_{aMP} / \sigma^2_{MP}$. However, if the β is being derived from a historical sample that includes non-systematic distortions an additional error term must be considered: $\beta_{€} = (\sigma_{aMP} + €) / \sigma^2_{MP}$. In the approach applied above, it should be noted that the error term is cumulative throughout the sample. That is, each month the prior month's adjusted stock price multiplied by 1.3 times the current change in the Index becomes the starting point from which a random number generator might then introduce as much as +/- 10% random noise to arrive at the new 'distorted' month-end price. More detail upon the construct of the addition of noise is given in Appendix One

Knowing that the underlying noise-free data was constructed with a pure 1.3 β the first salient observation should be the remarkable range of derived 'noisy' betas observed. The 0.77 β range (0.84 to 1.61) affords a significant opportunity to make the wrong inference about the underlying security's true β. The range of all the observations represents 59% of known β of 1.3. While the simple average of all 10 betas, at 1.19, comes quite close to the actual β, in the real world it would not be possible to re-process the historical prices on one single security 10 times and get a different answer each time. By definition, historical prices are fixed in time and amount. Also, it should be noted that the general level of R-Squared never rises above 0.40 in any of the simulations.

We can run the same scenario at the 2%, 5%, 10% and 15% noise levels:

TABLE B – Comparison of Historic Stock Price Regressions, with Random Noise added at Plus/Minus 2, 5, 10 & 15%

	Up to +/- 2% Monthly Noise			Up to +/- 5% Monthly Noise			Up to +/- 10% Monthly Noise			Up to +/- 15% Monthly Noise		
Instance	Observed β	R-Squared	Effective Annual ROR	Observed β	R-Squared	Effective Annual ROR	Observed β	R-Squared	Effective Annual ROR	Observed β	R-Squared	Effective Annual ROR
TEST A	1.333	0.889	10.9%	1.539	0.785	12.4%	0.964	0.188	5.6%	1.500	0.215	2.2%
TEST B	1.383	0.911	13.6%	1.280	0.613	11.5%	1.344	0.337	5.4%	1.674	0.248	10.7%
TEST C	1.284	0.915	15.5%	1.290	0.648	14.3%	1.248	0.299	6.4%	1.395	0.195	13.1%
TEST D	1.334	0.890	7.6%	1.316	0.672	7.2%	1.613	0.400	29.7%	1.529	0.250	20.7%
TEST E	1.286	0.885	12.7%	1.601	0.718	13.5%	1.290	0.282	-3.8%	0.934	0.112	9.5%
TEST F	1.297	0.892	15.7%	1.392	0.700	18.5%	0.896	0.204	7.7%	1.265	0.162	-6.1%
TEST G	1.285	0.912	13.9%	1.569	0.699	15.9%	1.137	0.245	18.1%	1.729	0.281	27.2%
TEST H	1.353	0.905	14.5%	1.149	0.586	13.1%	1.290	0.306	0.3%	1.636	0.237	11.4%
TEST I	1.370	0.914	15.0%	1.362	0.625	14.4%	0.840	0.206	-8.9%	1.341	0.173	19.6%
TEST J	1.355	0.920	11.4%	1.423	0.682	8.7%	1.273	0.331	3.0%	1.352	0.179	15.0%
Simple Average	1.328	0.903	13.1%	1.392	0.673	13.0%	1.190	0.280	6.4%	1.436	0.205	12.3%
Minimum Observed	1.284	0.885	7.6%	1.149	0.586	7.2%	0.840	0.188	-8.9%	0.934	0.112	-6.1%
Maximum Observed	1.383	0.920	15.7%	1.601	0.785	18.5%	1.613	0.400	29.7%	1.729	0.281	27.2%
Range	**0.099**	0.035	8.1%	**0.452**	0.199	11.3%	**0.773**	0.212	38.6%	**0.795**	0.169	33.3%

The effective (geometric) annual rate of growth of the S&P/TSX Index over the five year period under observation was 10.65%. Therefore the effective annual Rate of Return (ROR) for each of the stock price instances is provided here to give some indication as to how much random non-systematic noise has impacted the cumulative return of the stock. Mathematically it can be shown that, if (and only if) the period to period change in the index was uniform throughout the 60 months, then a perfectly correlated share with a Beta of 1.3 and no random noise added would be expected to show an effective annual rate of growth of 10.65% x 1.3 = 13.85%. However, this simple linear relationship cannot be expected to hold if the rate of change in the underlying market index varies from month to month (and it does). For practical purposes, one can expect that the ‘pure’ unaltered ROR of a 1.3 Beta share would have approximated a 13.5% to 14.0% annual return, and any outcome shown in the “instances” above different than this amount is a result of the cumulative random noise added.

OBSERVATIONS

Not surprisingly, there is a correlation between the size of the Range of β observed with the level of random noise added to the stock history. At the low-end of +/- 2% noise, the range was approximately 0.1 β. This ranged increased disproportionately until, at +/- 15% noise, it was approximately 0.8 β. Clearly, then, the larger the relative non-systematic price shifts are to any given security, the more likely that the regressed β developed from any single 60 month historic data sample will be farther away from the 'true' β.

Indeed, to examine how many of the 10 observed betas in each of the 4 noise classifications above fall within 10% of the 'true' β of 1.3 (i.e. $1.17 \leq \beta \leq 1.43$) we have:

	+/- 2%	+/- 5%	+/- 10%	+/- 15%
# of Observed Beta from sample of 10 within 10% of 1.3	10	6	5	4

So, even without putting any greater mathematical rigor into our decision making process than to expect the observed β to fall within 10% of the actual β all of the time, would mean that +/- 2% non-systematic noise would be the most amount we could allow[10].

If our standards were not as stringent, and we could accept an accuracy of an observed β that fell within 25% of the 'true' β of 1.3 (i.e. $0.975 \leq \beta \leq 1.625$), then:

	+/- 2%	+/- 5%	+/- 10%	+/- 15%
# of Observed Beta from sample of 10 within 25% of 1.3	10	10	7	6

Of course the point is that there is no manner of pre-determining, with any degree of reliability, how much non-systematic noise is in any actual historical stock price sample[11]. Using the simplistic small-scale example presented above, if one were to presume that the selected historical data only included a maximum of +/-2% monthly noise, the worst-case observation (i.e. the instance with the observed β farthest from the known true β of 1.3) would be Test Stock B where β = 1.383. On the other hand, if that presumption was nothing more than wishful thinking, and the actual (but unknown) level of monthly non-systematic noise was a maximum of +/-10%, the worst-case observation would be Test Stock I with a β of 0.84.

[10] Actually, to be a little more precise, the level of random noise added where all of the sample observations fall with 10% of a beta of 1.3 lies somewhere between +/-2% to +/- 5%. Moreover, a sample size of just 10 observations would not be considered statistically reliable – we would wish to increase the number of instances tested.

[11] After the regression, the R-Squared may provide a quantified measure of how well a straight line was fit to the stock prices. This, in turn, may allow a means of making an *inference* about how much noise is in the sample – but this inference is made on the assumption that the regressed beta is the correct one.

To put this kind of estimation error into context, if the MRP is assumed to equal 7.0% and the applicable R_f is 4.0%, then:

$$k_e = R_f + \beta \text{ x } (MRP) = 4.0 + 1.383 \text{ x } 7.0 = 13.681\%$$

vs.

$$k_e = R_f + \beta \text{ x } (MRP) = 4.0 + 0.84 \text{ x } 7.0 = 9.88\%$$

… a substantial difference, particularly when it is known, in these hypothetical simulations, that the 'true' β would require an expected return of:

$$k_e = R_f + \beta \text{ x } (MRP) = 4.0 + 1.3 \text{ x } 7.0 = 13.1\%$$

<u>QUESTION ONE</u>

i) *How sensitive is an analytically derived stock β to the distortions of non-systematic price fluctuations (and other 'noise' elements)?*

CONCLUSION i) **Regressed Betas are sensitive to non-systematic noise and become increasing more so as the potential variance of the noise increases.**

In pursuit of the second question:

<u>QUESTION TWO</u>

ii) *'Could the regression of any single stock price history be expected to yield a dependable estimate of the 'true' β?'*

CONCLUSION ii) **Our empirical observations of the hypothetical random noise data would probably lead us to conclude in the negative. The range of observed β expressed as a % of true β was either 7%, 35%, 59% or 61% depending upon the potential magnitude of the noise added (plus or minus a maximum of 2%, 5%, 10% or 15% respectively).**

Of course, if one could qualify that the historical prices remained largely unaffected by non-systematic impacts (e.g. each month-end price was within 5% of the undistorted price), then it would be reasonable to conclude that an acceptable range estimate of β would be obtained. As has already been discussed, however, it is just simply not possible to discern how much of the real-world historical price result is attributable to systematic vs. non-systematic impacts.

On this point some anecdotal observations might be made – but these are not supported analytically. It is likely that large multinationals that are conglomerates of highly diversified product lines are much more insulated from non-systematic distortions. The reason, of course, is that their net free cash flows are not dependent upon one geographic region, industry or market niche. It is more likely that negative non-systematic impacts incurred by one division in a given time period will be neutralized by positive results from another. Conversely, the highly specialized one-asset, one-product or specialty niche firm is likely to suffer from a greater volatility of non-systematic impacts shifting their stock price.

In consideration of the third question; which analytical techniques might be employed to improve the accuracy of the observed β – there is one particularly effective solution. And, while these may appear to only have application in the hypothetical world which we are currently considering (i.e. where we know the 'true' underlying β from the outset), there are, in fact, real world implications as will be discussed.

It has been shown that relying upon the regression outcome of any given single stock history may generate spurious results simply because of the non-systematic price impacts. A better approach would be to take a weighted average β of a homogeneous group of shares as a representative proxy for the β of the share issue that is under study. Using the +/- 10% noise level again as a representative example, note how the relative accuracy of the β estimate improves as a larger group of shares are regressed:

TABLE C - Groups of Averaged Regressions vs. Single Share Regressions

Instance	Beta Ranking	Observed β	R-Squared	RSq Ranking
TEST I	8	0.84000	0.20600	8
TEST D	7	1.61300	0.40000	1
Average of 10	6	1.19000	0.28000	7
Average of 20	2	1.29200	0.31700	6
Average of 30	5	1.34640	0.33882	2
Average of 40	3	1.33944	0.33599	3
Average of 50	4	1.34388	0.33480	4
Average of 100	1	1.29341	0.32711	5

While a single-share regression may have suggested a β as low as 0.84 or as high as 1.61 (Test I and Test D respectively), a weighted average of a group of 10 or more security names become much better estimators of β. Note, however, that the outcomes are not ordinal – that is, averaging more securities into the regression does not necessarily continuously improve the estimate[12]. For example, the average of 20 shares names at 1.292 β is a better predictor than those produced for groups of 30, 40 or 50 different securities.

[12] However, it is probably true that once the sample size is increased into the hundreds or thousands then the accuracy of beta predictions would continue to improve with every increase in sample size. This would be so because the sum total of the error term would tend towards zero as more and more data points were added. For practical purposes it would be unlikely that any homogeneous group of more than 50 shares could possibly be found (probably representing an entire industry).

Note, as well, that there is no correlation between the accuracy of the β prediction and the relative precision of the R^2 results. In fact, the best-fit line, as represented by an R^2 of 0.40 belongs to the seventh-place β estimate and the very best β estimate (average of 100 stocks) only has the 5th best R^2. These issues are discussed in greater detail in Accession paper "R-Squared and T-Statistics as Predictor of Beta Accuracy". Throughout this investigation we have, for simplicity relied upon the fact that the 'true' β for all the shares regressed has been entirely uniform at β =1.3. This would not, of course, be the case if a representative sample group of similar shares were selected to generate an average β. The issue of regressing a group of shares all with different 'true' betas is dealt with in an adjunct paper "R-Squared and T-Statistics as Predictor of Beta Accuracy".

QUESTION 3

iii) Are there any analytical techniques that can be applied in the data collection or processing phases that tend to improve the reliability of the derived β?

CONCLUSION iii) Weighted averaged[13] betas from groups of stocks that are reasonably close proxies for the single share issue under investigation will generally result in a more accurate prediction of the 'true' composite β rather than relying upon the outcome of one single share regression.

Suppose that one is attempting to derive the β of a mid-cap microchip manufacturer. Conclusion ii) would cause us to become suspicious of the accuracy of the historical regression for that security alone. The real-world application of Conclusion iii) should suggest that a much more reliable estimate could be obtained by selecting 10 or 20 similar publicly-traded enterprises that can be found in that industry from which to generate an aggregate β. While it is true that none of the proxy firms would face precisely the same risk-profile as the subject firm (and therefore would all have different betas), this probably would not matter much if the subject were representative of the industry average. Of course, if the subject firm is a unique player dissimilar to every other firm in the industry, then no statistical technique will be suitable in predicting a representative cost of capital anyway. This method of constructing a weighted average composite β has the benefit of minimizing the distortions of non-systematic noise because the residual errors of each security tends to be offset or mitigated by the error terms in the other stocks[14].

[13] Since all the shares price paths generated in this random model began at $25 and, in their unadjusted form, changed in precise step with the S&P/TSE300, the weighting required was just a simple average (i.e. in the group of 10 equities, each beta observation was weighted at 10% and, in the group of 20, they were weighted at 5% each, etc.). In the real world, this would not be the case and what adds complexity here is that each month-end market cap upon which the relative weighting would be derived also includes the share appreciation/depreciation from the non-systematic impacts.

[14] Since the error-terms (noise) added is random and normally distributed, the larger the sample size, the more the sum of the error-terms would be expected to tend towards zero.

APPENDIX ONE - METHOD OF ADDING NOISE TO UNADJUSTED STOCK PRICE:

Assuming a 'noise-free' or **Unadjusted Stock Price** with a β of 1.3:

Unadjusted Stock Price = (1+ (Δ_m x 1.3)) x Prior Month's Unadjusted Stock Price

Where

Δ_m = % month-over-month change in S&P/TSE Index

Arbitrarily the initial Unadjusted Dec. 31, 2001 Stock Price has been set to $25.00

Now a monthly rate of +/- 10% of noise is added:

Adjusted Stock Price =

(1+ (Δ_m x 1.3)) x Prior Month's **Adjusted** Stock Price x Є

Where

($0.9 \leq$ Є ≤ 1.1) and Є is selected by a random number generator

Such a method of arriving at the Adjusted Stock Price is believed to be consistent with what might actually happen as a result of a non-systematic price shift. Consider, for example, a manufacturing firm experiencing only systematic share price changes until an industrial accident forces it to temporarily close one of it's plants. At that point a non-systematic price shift downward reflects the expected one-time economic loss of the plant closure. Subsequently, in the absence of any further non-systematic shocks, the stock will continue to track the market index from the new lower level.

TECHNICAL POINT ON BETA REGRESSION SHORTCUT:

The method of regressing historical stock prices in this paper (and many other financial references) is not precisely true to the CAPM theory, but yields the same results. Here we describe a share with, for example, a Beta of one, moving in precise step with the market index. That is, if the month-over-month market index increases 2% then the relative month-over-month appreciation in the share price is also expected to be 2%.

In reality, Beta measures the relative change in the security to the market movement ABOVE THE RISK-FREE RATE OF RETURN (i.e. just the Market Risk Premium). Implicitly both the market and the equity in question are expected to always deliver above the risk-free rate of return (else why would any risk-adverse investor put their funds at risk?). So, in the 60 month S&P/TSE return referenced in the paper a 10.65% annual return was observed. This would equate to an effective (i.e. geometric) monthly average return of 0.8469%. If it is assumed that 4.0% was the appropriate annual risk-free rate, then this would equate to a 0.3274% ($1.04^{(1/12)} - 1$) monthly return that a shareholder must earn just to compensate them to forego the risk-free rate.

What this means is that, if the month-over-month market index increases 2%, it is only the 1.6726% (2.0% - 0.3274%, as the first 0.3274% return just represents investor compensation for the risk-free rate) change that should be considered as the delta X in the regression. Similarly, all the resultant delta Y's should be similarly reduced by a monthly 0.3274% as these also would be expected to consistently return at least the risk-free rate. Because the same constant is being subtracted from both the X and Y data points it can be shown, therefore, that the regressed Beta that results will be the same as if this additional step were not taken. As a convenience, then, most financial descriptions of the covariance of market-to-share-price changes state that a total 2.0% change in the market will result in a β(2.0%) change in the share price. Whereas the more precise relationship of the two would actually be ($2.0\% - R_f$) equates to an expected $\beta(2.0\% - R_f)$ change in the share price.