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MEASURING THE ERROR OF ESTIMATION IN GROUPED STOCK PRICE BETAS

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EXECUTIVE SUMMARY:

- The paper is the conclusion of a three-part series
- The subject series has been how to indentify, measure and mitigate the effects of non-systematic price impacts to stock beta regressions
- Individual firm betas are probably unreliable, but the reliability can be improved by grouping comparable firms into an aggregate index
- The impact of adding random noise to a sample of 30 individual "un-distorted" firm betas is measured
- Combining the 30 distorted price changes into one aggregated index greatly mitigates the impact of the noise which is randomly distributed.

INTRODUCTION & OVERVIEW

This paper is the third in a series dealing with the distortive impacts of non-systematic price variances to standard stock beta regressions. The difficulty in applying the Capital Asset Pricing Model (CAPM) to 'real world' cost-of-capital investigations always has been that real world historical data is tainted by non-systematic (or firm specific) price impacts. The first paper in this series, "Sensitivity of a Stock Beta to Non-Systematic Price Impacts" showed that even relatively small infusions of non-systematic price changes causes significant inaccuracies in the regressed beta and resulting cost of equity. **Most importantly, the first paper concluded that the beta regressed from any single stock price data was probably wholly unreliable as a result of non-systematic price distortions, except by pure chance**. The second in the series "R-Squared and T-Statistics as a Measure of Stock Beta Accuracy" considered if the impacts of price distortions could be measured. That paper concluded that common statistical procedures would only measure the linearity of a stock price movement with the market index – there is no effective means of segregating the systematic from non-systematic price effects.

Finally, this paper, as a conclusion to the series, will focus upon the question as how to minimize the potential impact of non-systematic price changes upon regressed betas. The methodology follows that employed in earlier papers: obtain a hypothetically "un-distorted" sample of individual firm betas; on a random basis over a 60 month period add up to plus or minus 10%/month[1] of non-systematic price changes; compare the betas regressed from the randomly 'distorted' data with those obtained from the systematic or "undistorted" pure data.

In the first paper that applied this methodology, the groups of firms in a sample were presumed to all have exactly the same starting "pure" beta and market capitalization. This simplification was necessary to quantify the range of distorted betas that would be observed given the addition of a certain level of 'noise' (i.e. distortion). In this paper, in order to simulate real world applications of the proper grouping methodology, we begin with a sample of 30 firms that has a wide range of market capitalizations and starting betas. This data is obtained from a subset of firms in the New York Stock Exchange (NYSE) Energy Index. The method in which the sample was obtained is detailed in Appendix One.

The reader is presumed to have a solid understanding of CAPM and linear regression. The body of the paper attempts to remain at a conceptual level. A technical familiarity with the Least Squares Methodology, Residuals, Standard Errors and the components of the R-Squared Ratio will be required, however, in order to fully appreciate the pragmatic benefits of grouping a sample of comparable firm price movements in order to derive a

[1] As explained in the appendix to the first paper, adding up to 10% non-systematic distortion means that, if a stock had, for example, a "pure" beta of 1.3, then, in a month where the Market Index ascended 1.0%, that stock would be expected to increase 1.3%. However, with up to +/- 10% distortion, that stock would be expected to randomly ascend anywhere in the range of [(1 + (1.0% x 1.3))w – 1]. Where w is a randomly generated number: 0.9 </= w </= 1.1. Therefore the monthly combined (systematic + nonsystematic) stock appreciation could range anywhere between -8.83% to +11.14%

reliable beta proxy for a subject firm. These statistical measures are reviewed in Appendix Two and each is displayed in a graphical context. For the reader not as familiar with the statistical terminology, it is important to keep a visual image of the distances being measured from the regression line to the data point in mind.

THE BETA SAMPLE

We hope to develop a reliable beta for the subject firm. If that firm is public, we can regress the historical data and observe the beta that results from that. However that single firm beta is probably distorted with non-systematic price shifts. Often, too, the R-Squared that results from a single firm regression is so low as to warrant the regression results as unusable. One possible solution is to identify as many likely comparable firms as proxies for the subject. In combining the price movement data of the group, the non-systematic distortions are largely neutralized, as this paper will attempt to show. It will still be necessary, however, to begin with statistically reliable data for the group. That means the individual R-Squared of the group constituents must be above a predetermined minimum before being selected for inclusion in the group. This aspect is discussed in greater detail in Appendix One.

The reader is referred to the Appendix One Table "INDEX COMPOSITION DATA FOR NYSE ENERGY INDEX AS OF 04/04/2008" to consider the thirty different firms in the selected sample, along with their individual betas and market capitalizations. The summary of that data is reproduced here:

	Beta	Market Cap in Billions USD
Simple Average Beta	1.10	17.94
Range	0.94	33.95
Standard Deviation	0.21	10.34
Minimum	0.56	5.24
Maximum	1.50	39.19
Beta weighted by Market Capitalization	1.06	

Our subject firm lies within the $15B to $20B market capitalization range. So the average of our sample at almost $18B represents this well. However the range of market caps in the sample is quite wide ($5 to $40B) and we would have preferred this to be more narrow – but there was just not sufficient depth of data available in the Energy Index.

The reported betas have been obtained directly from the NYSE and we are going to hypothetically presume that these betas are "true". The Betas also represent quite a wide range of (from 0.56 to 1.50) of observations and some of the reasons for this is discussed in Appendix One. For our experiment, the wider range of betas suits our purposes as the non-systematic variability will be increased by the larger risk gamut. Note that in its assumed "pure" state, the simple average beta for the group is 1.10 and the weighted average beta is 1.06

ADDING RANDOM NON-SYSTEMATIC DISTORTIONS

Upon adding the non-systematic price impacts as described, the nature of the 30 individual betas changes significantly:

Sample Firm #	Instance	"True" β	Observed β	Beta Difference	R-Squared	Absolute Sum of Residuals for all 60 Months	Standard Error of the Estimation
1	TEST A	1.010	0.741	-0.269	**0.081**	320.82%	0.39%
2	TEST B	0.870	0.675	-0.195	**0.064**	338.45%	0.42%
3	TEST C	1.300	1.318	0.018	0.228	318.47%	0.37%
4	TEST D	1.030	1.380	0.350	0.275	271.34%	0.32%
5	TEST E	1.000	1.796	0.796	0.337	328.94%	0.40%
6	TEST F	1.060	0.698	-0.362	**0.089**	288.36%	0.31%
7	TEST G	1.300	1.523	0.223	0.306	289.64%	0.33%
8	TEST H	0.740	0.240	-0.500	**0.009**	320.30%	0.40%
9	TEST I	1.350	1.569	0.219	0.332	277.91%	0.31%
10	TEST J	0.930	0.675	-0.255	**0.067**	318.54%	0.40%
11	TEST K	0.870	-0.012	-0.882	**0.000**	312.64%	0.39%
12	TEST L	0.560	0.637	0.077	**0.064**	315.88%	0.37%
13	TEST M	1.090	1.355	0.265	0.229	315.96%	0.39%
14	TEST N	1.130	1.053	-0.077	0.130	364.32%	0.47%
15	TEST O	0.900	0.681	-0.219	**0.079**	306.49%	0.34%
16	TEST P	1.110	1.174	0.064	0.188	311.54%	0.38%
17	TEST Q	1.160	1.110	-0.050	0.162	324.67%	0.40%
18	TEST R	1.160	0.998	-0.162	0.136	314.08%	0.40%
19	TEST S	1.450	1.518	0.068	0.291	305.36%	0.35%
20	TEST T	0.990	1.026	0.036	0.167	277.52%	0.33%
21	TEST U	1.140	1.186	0.046	0.172	337.75%	0.43%
22	TEST V	0.950	1.041	0.091	0.159	312.45%	0.36%
23	TEST W	1.220	1.277	0.057	0.204	326.73%	0.40%
24	TEST X	0.900	1.129	0.229	0.157	339.37%	0.43%
25	TEST Y	1.190	1.111	-0.079	0.172	316.48%	0.38%
26	TEST Z	1.410	1.730	0.320	0.378	273.30%	0.31%
27	TEST AA	1.500	1.267	-0.233	0.195	340.35%	0.42%
28	TEST BB	1.290	1.002	-0.288	0.169	277.20%	0.31%
29	TEST CC	1.190	0.835	-0.355	0.107	318.58%	0.37%
30	TEST DD	1.270	1.572	0.302	0.280	323.79%	0.40%
Simple Average		1.102	1.077	-0.026	0.174	312.90%	0.40%
Minimum Observed		0.560	-0.012	-0.882	0.000	271.34%	0.31%
Maximum Observed		1.500	1.796	0.796	0.378	364.32%	0.47%
Range		0.940	1.808	1.678	0.378	92.98%	0.16%

Now the range of the observed beta have doubled from 0.94 ‘true’ to 1.808 ‘distorted’. The individual R-Squared less that 0.10 (i.e. the regression has fit less than 10% of range of data) have been highlighted in red. If this were real world data, we would be inclined to discard the firms with $R^2 < 0.10$ as being statistically unreliable. However, the sample data shall be retained as is in order to show that, even with the inclusion of 8 non-linear firms, a reasonable approximation of the original 'pure' beta can be recreated through the correct grouping methodology.

The above distorted betas have been regressed against the S&P500 index for the sixty months of April 2003 through March 2008 inclusive. Over that period of time, the simple average monthly change for the S&P500 was a positive increase of 0.63% Consider, then, that if a regressed stock of beta = 1.0 happened to have a Least Squares line fit such that each month it continuously averaged a 1.0% positive increase, the sum total of all the absolute residuals would then amount to (1.0% - 0.63%) x 60 months = 22.2%. (for a complete description of the Residual totalistic, see Appendix Two. Instead the average absolute sum of all the residuals for our entire sample is 312.90%. In any given month, then, the average distance between the fit regression line and the actual data point would equate to approximately 5.2% (312.90 / 60). Recall that, in a perfect regression, where all the data points fell exactly on the fit line, R^2 = 1.0 and both the sum and average monthly residual would be the same; 0.0%

So it should be clear then, that our experiment has been quite effective in adding considerable variability to the data around the fit regression line. The 30 distorted betas above are truly noisy and the highest R^2 reported is less that 0.38

GROUPING THE SAMPLE

There are essentially two methods of grouping the sample data such that a reliable average beta can be developed. The first is to disregard the market capitalizations of the respective 30 constituents and treat each firm with the same import as every other in the sample. Such an approach would be appropriate if all the firms in the sample had a narrow range of market capitalizations, with an average close to the market cap of the subject firm.

Each month-end index of averaged rates of stock price changes is constructed via:

$[\Sigma (M_{it} / M_{i(t-1)}) - 1] / n$

Where:

M_{it} is the Market price of stock i at month-end t
$M_{i(t-1)}$ is the Market price of stock i at month-end prior to period t
n is the number of stocks in the sample

This process is repeated at each month-end. Once a sixty-month index of simple average stock delta's is compiled, this data is regressed against the Market Index in order to determine the new 'grouped' beta. Applying this method to our random data as previously described yielded the following results:

GROUPED BETA RESULTS – SIMPLE AVERAGE

Grouped Beta	1.077
R^2	0.8665
Sum of Absolute Residual Errors	47.71%
Standard Monthly Error of the Estimation	0.01%

Note the substantial increase in R-Squared. Where the average of the 'distorted' group of 30 R-Squared's had amounted to no more than 0.174, the combined regression of the thirty stock price movements yield an R-Square of 0.866. The reason why is because the random noise add to each of the thirty monthly share movements would be normally distributed. That is, it would be just as likely for a non-systematic price impact to cause a stock to move above it's 'true' beta value as it would be to move below. Hence, if the thirty monthly price movements are first combined into one, and then regressed, the random noise tends to cancel it self out. This impact can clearly be seen is the fact that the sum of the absolute residuals for all thirty shares amounted to almost 313% for the 'distorted' individual betas, but only 48% for the combined group. The process of combining the individual price movements into one representative index has reduced the unexplained distance around the regression line to less than $1/6^{th}$ of its previous level.

In the event that the comparable firms in the sample are not tightly distributed in terms of overall market capitalization, it may not be appropriate to use a simple average of all the stock price movements. This is because the well-observed Size Effect on cost of capital. Smaller firms, all other things being equal, have higher costs of capital than do larger. Correspondingly this also means that smaller firms have higher betas. The 30 firm sample we selected from the NYSE Energy Index ranges in market cap from $5B to $40B and those at the extremes will not have the same risk-profiles as those closer to the mean. In cases like these, the grouped index will maintain greater fidelity with the group as a whole if the data is first weighted by market capitalization relative total market capitalization of the entire sample.

But when to measure the market weighting? There are, in general, three possible approaches:

1. A continuous weighting, that re-measures the market cap of each of the sample firms at each month end and apportions the month-over-month stock price changes accordingly
2. An approximation of the above. For example, finding the 60 month sample beginning and ending market capitalization values and using an average of the two.
3. A single point weighting. For example, using either the 60 month starting, midpoint or ending market capitalizations.

Theoretically, the most justifiable approach is the first. The non-systematic price impacts are embedded in the monthly price delta's. Any other method of apportioning the weighting of these distortive impacts would be purely arbitrary and further increase the distortion. Since the purpose behind grouping the data in the first place is that the random noise tends to cancel, it makes sense that the amount of distortion aggregated into the whole remains in proportion to the actual noise incurred.

Depending upon the sample size, however, the relative difference between the three difference approaches may not be material. The difficulty here is that it is virtually impossible to know this until all three calculations have been completed.

GROUPED BETA RESULTS – WEIGTHED AVERAGE

Method One – Continuous Month-End Weighting:

Grouped Beta	1.115
R^2	0.8222
Sum of Absolute Residual Errors	62.57%
Standard Monthly Error of the Estimation	0.02%

Method Two – Approximate Weighting

At the start of the 60 months, the total market capitalization for all thirty firms in the sample is $538.140B and, at the end of the 60 months it is $695.525B. The proportionate amount that each stock is of both the beginning and ending market value will be determined and those two numbers will be averaged. It is this constant average proportion for each share, that will represent how much of that share's net monthly change is included into the overall combined index.

Grouped Beta	1.081
R^2	0.8353
Sum of Absolute Residual Errors	56.70%
Standard Monthly Error of the Estimation	0.01%

Method Three – Single Point Weighting; End of Period

On the premise that all such regression exercises would be completed using historical data, some argument could be made for using the period-end (i.e. the 60th month) market capitalization weightings. Proponents of this view would state that these weightings are the most current, and therefore the most reflective of future expectations with respect to how this sample will be constituted in the immediate future and beyond[2].

Grouped Beta	1.133
R^2	0.8348
Sum of Absolute Residual Errors	59.65%
Standard Monthly Error of the Estimation	0.02%

[2] The problem with this perspective is that each stock's beta is regressed from historical data during which the risk profile of the firm, relative to the market, was not static. If, for example, in the 60 month regression data a small firm became a very big firm, it's moving 60 month average beta over that time would be expected to have decreased, ceteris paribus. This would be an example of the Size Effect.

OBSERVATIONS ON BETA GROUPING METHODS

Weighting Methodology Applied	Grouped Beta	R-Squared	ABS Res. Errors		Assumed K_e if r_f=4.0% & MRP = 7.0%
Simple Average of Monthly Change	1.077	0.8665	47.71%		11.54%
W. Avg: Weight revised at each month-end	1.115	0.8222	62.57%		11.81%
W. Avg: Weight is avg of Month 0 & 60	1.081	0.8353	56.70%		11.57%
W. Avg: Weight is month 60 proportions	1.133	0.8348	59.65%		11.93%

The table above summarizes the output generated from the four different grouping methods applied. In addition, the right-most column reports what the resultant CAPM cost of equity capital would be under each scenario, assuming a 4.0% risk-free rate and 7.0% MRP. As a general observation, there is no real material difference in outcomes. Few valuations would be so precise or so sensitive to discount rate such that a 39 basis-point difference would be considered significant. However, these results are probably indicative of the relatively large sample size of 30 firms. If it were possible, for example to only obtain 10 comparable firms for the sample, it is likely that the grouping method applied would have had a much greater impact on the final results.

If R-Squared were the only criteria upon which to select the weighting methodology, the simple average outcome would be the preferred choice from the alternatives available in this case. As illustrated in the very first paper in this series, however, R-Squared is only a measure of the quality of the fitted-line, not a predictor of fidelity with the 'un-distorted' data. Theoretically the Weighted Average method, using revised weights continuously undated at each month is the most sound[3]. In the absence of any evidence to the contrary or special overriding circumstances, the 1.115 beta outcome would be the preferred choice.

[3] If, for example, the firms in the sample are incurring rapid continuous changes in capitalization over the period of observation, the only way to ensure their betas are proportionately represented in the combined monthly data is to "re-balance" the weighting factors at each month-end.

REVIEW & CONCLUSIONS

- We required a beta for a subject firm and knew a single-firm regression was probably distorted and unreliable
- We indentified a sample of 30 comparable firms
- Hypothetically, for the purposes of this experiment, we accepted the reported betas for these firms as “pure”, “undistorted” and randomly added non-systematic noise to each
- The hypothetically “undistorted” average beta for the sample ranged from 1.06 to 1.10
- After noise was added, the single firm betas ranged from a marginally negative amount to 1.8
- The individual average R^2 of the distorted sample was 0.174
- By combining the month-over-month relative price changes of the sample group into one aggregated whole prior to conducting the regression, the R^2 was increased to the high 0.82 to 0.88 range and beta determined to be 1.11

It may seem ironic, that an effective means of eliminating non-systematic price distortions in a single-firm beta is by adding more instances of distorted data to it. On the premise that non-systematic price impacts will be normally distributed, it is likely that the negative price impact on one stock will be counteracted by a contemporaneous positive price effect upon another stock in the combined group. Empirically this ‘cancelling of noise’ was demonstrated in the foregoing experiment via the dramatic reduction in estimation error (the physical distance of the residuals) once the sample firms had been combined. This concept is in keeping with an underlying theorem of the CAPM itself which states that non-systematic risk is entirely diversifiable if a sufficiently broad basket of the market asset is held.

We have, therefore, found a reasonably effective method of mitigating the distortion caused by non-systematic price impacts in individual stock betas. We have not, however, determined what the "true" beta of the subject stock would be. This might only be done if there was some practical means of identifying the components of systematic vs. non-systematic price changes. Instead, we have just constructed a proxy beta for what the "true" individual beta might be. The validity of this approach, however, depends entirely upon how closely comparable the firms in the sample are (either individually, or as a composite in aggregate) to the subject firm.

APPENDIX ONE

METHODOLOGY FOR CONSTRUCTING A HYPOTHETICAL GROUPING OF STOCK BETAS

With the intent of building a representative group of proxy firms in a given industry with which to derive a 'distortion-neutralized' beta, the following methodology has been employed. At the outset it was decided that the group sample size should be thirty firms. Thirty is a statistically significant number, while still being small enough to keep the presentation of all the data from becoming onerous. It would have been possible to randomly create starting "true" betas and market capitalization numbers to each of the thirty constituents, however that approach would have lacked a certain harmony with 'real world' circumstances.

Instead, in order to capture a realistic range of betas that would be typical of a given industry, a sample industry was selected (the Energy Industry) and, to make the experiment even more practical, it was decided that the sample group should reflect a hypothetical firm for which a beta proxy was too be developed. The hypothetical firm is to be an integrated Oil and Gas company, with a current market capitalization in the range of $15 to $20 billion.

The NYSE Energy Index (comprised of a wide variety of 145 constituent companies) was selected as the general population. From that, the desired sample of 30 firms were selected. Of the 145 firms in the population, the NYSE had only published a Beta for 114 of the total population, so 31 need to be immediately disqualified[4]. In the remaining constituent, reported Betas ranged from 0.29 to 1.99 and Market Cap from $0.15B to $480.71B. While our hypothetical subject firm may eventually be calculated to have virtually the same beta as those firms at both the high and low market cap of the Energy Index spectrum, this would only occur by chance. There is little point in comparing a $15B market cap firm with neither one of $150B nor one of $0.15B. The Size Effect,

[4] It was decided that ONLY NYSE Beta references were to be used in order to eliminate differences in measurement practices.

stating that on average smaller firms require higher costs of capital, has consistently been observed to be valid. Therefore, we need to eliminate those of the remaining 114 firms with market capitalizations that are not relevant to our $15 - $20B subject firm.

In theory, if we could identify the entire subset of firms with market caps in the $15 - $20B range and then randomly select 30 - that would suit our purposes perfectly. Unfortunately, that would only leave us with a sample of five firms. So we expand our selection criteria to include those firms with market caps greater than $5B and less than $40B. That provides a subset of 41 firms. From this subset, 11 are randomly eliminated, thereby giving us the following sample:

Co. # in 41 Subset	Co. # in Original 145	INDEX COMPOSITION DATA FOR NYSE ENERGY INDEX (SYMBOL NYE.ID) AS OF 04/04/2008 NAME	TICKER	COUNTRY	ICB	SUB SEC	Beta * as at Apr 4/08	Market Cap (in B USD)
1	17	Canadian Natural Resources Ltd.	CNQ	Canada	533	Exploration & Production	1.01	39.19
2	18	Halliburton Co.	HAL	United States	573	Oil Equipment & Services	0.87	36.65
3	19	Marathon Oil Corp.	MRO	United States	537	Integrated Oil & Gas	1.30	34.48
4	20	XTO Energy Inc.	XTO	United States	533	Exploration & Production	1.03	32.82
5	21	EOG Resources Inc.	EOG	United States	533	Exploration & Production	1.00	31.49
6	22	Anadarko Petroleum Corp.	APC	United States	533	Exploration & Production	1.06	31.01
9	27	Weatherford International Ltd.	WFT	United States	573	Oil Equipment & Services	1.30	25.95
10	32	Chesapeake Energy Corp.	CHK	United States	533	Exploration & Production	0.74	25.12
11	29	National Oilwell Varco Inc.	NOV	United States	573	Oil Equipment & Services	1.35	23.63
12	33	Baker Hughes Inc.	BHI	United States	573	Oil Equipment & Services	0.93	22.59
13	34	Petro-Canada	PCZ	Canada	537	Integrated Oil & Gas	0.87	22.37
14	36	TransCanada Corp.	TRP	Canada	577	Pipelines	0.56	19.60
15	35	Talisman Energy Inc.	TLM	Canada	533	Exploration & Production	1.09	19.26
16	52	Diamond Offshore Drilling Inc.	DO	United States	573	Oil Equipment & Services	1.13	17.17
18	38	Murphy Oil Corp.	MUR	United States	537	Integrated Oil & Gas	0.90	16.20
19	40	Noble Corp.	NE	United States	573	Oil Equipment & Services	1.11	14.23
20	41	Smith International Inc.	SII	United States	573	Oil Equipment & Services	1.16	14.11
21	42	Noble Energy Inc.	NBL	United States	533	Exploration & Production	1.16	14.00
22	43	Southwestern Energy Co.	SWN	United States	533	Exploration & Production	1.45	12.57
24	44	El Paso Corp.	EP	United States	577	Pipelines	0.99	11.89
26	46	Cameron International Corp.	CAM	United States	573	Oil Equipment & Services	1.14	10.10
27	48	Nabors Industries Ltd.	NBR	United States	573	Oil Equipment & Services	0.95	9.72
29	50	BJ Services Co.	BJS	United States	573	Oil Equipment & Services	1.22	8.73
30	51	FMC Technologies Inc.	FTI	United States	573	Oil Equipment & Services	0.9	8.02
31	53	Denbury Resources Inc.	DNR	United States	533	Exploration & Production	1.19	7.64
33	56	Grant Prideco Inc.	GRP	United States	573	Oil Equipment & Services	1.41	6.60
36	59	Sunoco Inc.	SUN	United States	533	Exploration & Production	1.50	6.29

38	68	Quicksilver Resources Inc.	KWK	United States	533	Exploration & Production	1.29	6.03
39	61	Cabot Oil & Gas Corp.	COG	United States	533	Exploration & Production	1.19	5.44
41	64	Helmerich & Payne Inc.	HP	United States	573	Oil Equipment & Services	1.27	5.24

* Betas as reported by the New York Stock Exchange

MARKET CAP TOTAL		538.14
Simple Average Beta	1.10	17.94
Range	0.94	33.95
Standard Deviation	0.21	10.34
Minimum	0.56	5.24
Maximum	1.50	39.19
Beta weighted by Market Capitalization	1.06	

We are presuming that our hypothetical subject firm is a fully integrated Oil and Gas play that operates business segments in Exploration, Production, Pipelines/Transportation, and Third-Party O&G Services. Therefore, it is appropriate to believe that the overall target firm beta will be a blended average of both the integrated comparables that similarly do all of those things, as well as the specialty firms that focus upon one or more of those segments. If this were not true, and our hypothetical subject firm only conducted Exploration and Production, for example, it would have made sense to narrow our population to only these types of comparables (ICB = 533)

The foregoing process has given us an unbiased means of selecting a group of 30 betas that are truly reflective of those actually being reported[5] in the Oil and Gas sector. We will use these as the starting point of our experiment.

At this point, in an actual application of compiling a comparable beta sample in order to construct a composite proxy for our subject firm beta, we would conduct regressions upon all 30 of the selected firms in order to test the statistical reliability of the individual betas:

[5] It should be remembered, however, that these individual actual betas are comprised of a blend of the systematic and all the non-systematic distortions that we have been attempting to measure throughout the series of these three papers. However, for the purpose of this experiment, we presume that the 30 betas contained in the sample are distortion-free non-systematic only betas to which we then add distortion and measure the effects.

NAME	R-Sqrd, 3YR Regression	R-Sqrd, 5YR Regression
Canadian Natural Resources Ltd.	0.170	0.117
Halliburton Co.	0.112	0.101
Marathon Oil Corp.	0.202	0.166
XTO Energy Inc.	**0.012**	**0.050**
EOG Resources Inc.	**0.015**	**0.015**
Anadarko Petroleum Corp.	0.159	0.129
Weatherford International Ltd.	0.132	**0.064**
Chesapeake Energy Corp.	**0.021**	**0.039**
National Oilwell Varco Inc.	**0.075**	**0.089**
Baker Hughes Inc.	0.241	0.166
Petro-Canada	0.244	0.149
TransCanada Corp.	**0.082**	0.103
Talisman Energy Inc.	0.328	0.204
Diamond Offshore Drilling Inc.	0.131	0.121
Murphy Oil Corp.	**0.040**	**0.023**
Noble Corp.	0.207	0.168
Smith International Inc.	0.113	**0.075**
Noble Energy Inc.	0.136	0.141
Southwestern Energy Co.	**0.059**	**0.060**
El Paso Corp.	**0.039**	**0.076**
Cameron International Corp.	0.241	**0.081**
Nabors Industries Ltd.	**0.043**	**0.074**
BJ Services Co.	**0.053**	**0.027**
FMC Technologies Inc.	**0.044**	**0.050**
Denbury Resources Inc.	**0.066**	**0.054**
Grant Prideco Inc.	**0.041**	**0.073**
Sunoco Inc.	0.226	0.169
Quicksilver Resources Inc.	**0.060**	**0.053**
Cabot Oil & Gas Corp.	**0.064**	**0.049**
Helmerich & Payne Inc.	**0.038**	**0.070**

The NYSE reports Beta's calculated on a weekly basis, using 3 years of historical data. The R-Squared statistics reported above are calculated using monthly data, and both 3 and 5 year regressions of historical data have been presented. The stock price movements have been regressed against the S&P500 Market Index in keeping with the

NYSE practice. Note that just about half of the R-Squared observations have values less than 0.10 (highlighted in red). This means that the regression, in these cases, only explained less than 10% of all the actual stock price distance away from the regressed line [for a visual representation of what the R-Squared statistic is actually measuring, see Appendix Two].

In an actual application, we would probably be prudent to reject these observations from our sample[6], and either decrease the acceptable sample size or attempt to find other substitute comparables with better-quality regressions.

For the purposes of this experiment, however, we are *assuming* that each of the beta's reported for these 30 stocks is the "true" un-distorted beta. The only purpose of collecting this sample of firm-betas from the NYSE was to obtain a realistic collection of industry beta values and market capitalizations. Since we will not be using the actual historical price data from the aforementioned 30 firms, there is little point in searching for better quality comparables.

MARKET CAPITALIZATION WEIGHTING

In an actual application of compiling a beta proxy, the market capitalization of the comparable firms may have a significant impact upon the overall construction of the proxy.

As a vast simplification of the weighting process, there are basically two general approaches that can be applied:

i) A Simple Average of each Beta in the Sample: The approach presumes that each of the comparable firms in the sample is equally representative

[6] It is not unusual for the majority of individual firm betas in any randomly selected sample to have very low R-Squared values. The critical question is, 'at what point should the lower level of R-Squared acceptability be set?' As yet there has been very little industry guidance provided on this key issue. We are arbitrarily suggesting a 0.10 limit herein, but can offer no quantifiable justification for this number.

of the subject firm. Therefore, it is appropriate for each constituent beta to be given equal weighting in the combined whole. This method is more suitable when the sample capitalization range is narrow and largely in agreement with the market capitalization of the subject firm.

ii) A Weighted Average Beta, Proportionately Based Upon the Market Capitalization of each firm in the Sample, relative to the Total Market Capitalization of the entire Sample: This approach presumes that no single firm in the sample is exactly representative of the subject firm, but that the aggregated whole is a true proxy for the subject. Accordingly, each beta in the group is included only to the extent that it proportionately represents the total sample market cap.

Note that approach ii), Weighted Average, also raises the issue of when the market capitalizations should be measured? Using a five-year regression period, for example, the proportionate market capitalizations at the start of the sixty months will certainly not equal the same proportions at the end of the regression period. This question is dealt with in greater detail within the body of the paper itself.

APPENDIX TWO

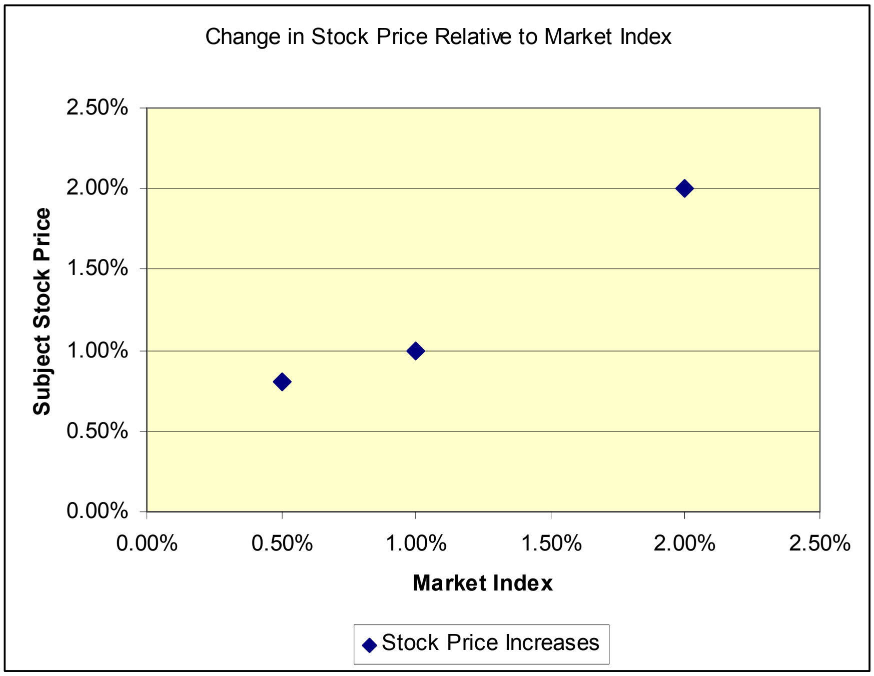
REVIEW OF METHOD OF LEAST SQUARES, RESIDUALS, STANDARD ERRORS AND R-SQUARED

In these series of papers we have been attempting to explain how a change in the Market Index (measured on the X Axis, the independent variable), is reflected in a change in the Subject Stock Price (measured on the Y Axis, the dependent variable). If the relationship by which Y changes in response to a unit change in X is linear, then the ratio between rates of change is known as Beta (β). If, for example, a subject stock is believed to have a 1.0 β then a 1.0% monthly increase/(decrease) in the Market Index would be expected to exactly correspond with a 1.0% monthly increase/(decrease) in the subject stock price.

METHOD OF LEAST SQUARES: Linear Regression is a procedure by which the relationship of a change in y is predicted to be explained by a change in x. The most common form of linear regression is the 'Least Squares' method. This procedure mathematically determines the smallest value of the sum of the predicted value of y's compared with the actual observed value of y's. A simple example will greatly enhance clarity.

Suppose at the end of one month we note that the Market Index has climbed 1.0% and the subject stock prices has correspondingly increased 1.0%. At the end of the subsequent month the Market Index has climbed 2.0% and similarly the subject stock has also ascended 2.0%. At this point there appears to be a direct and exactly proportional relationship between the The Market (x) and The Stock (x). This would imply that β = 1.0. Now, at the end of the third month the Market Index ascends 0.5% but the subject stock increases 0.8%. Our best estimate of β would no longer be 1.0 and the data can be represented graphically as:

	Month		
	1	2	3
Index	1.00%	2.00%	0.50%
Stock	1.00%	2.00%	0.80%



The term "y_i" generically represents the actual observed values of y at the ith observation in the sample. In the foregoing data, therefore, $y_1 = 1.0\%$, $y_2 = 2.0\%$ and $y_3 = 0.8\%$.

Recalling that the formula for any line is: $y = \alpha + \beta x$, Linear Regression will "fit" a line to any given set of x,y data coordinates. This "fitting" process is represented by the formula: $\hat{y}_i = \alpha + \beta x_i$ where $\hat{y}$ is the estimated value of y (as opposed to the actual observed value). The method of least squares determines that the best "fit" for such a linear approximation occurs when $\Sigma (y_i - \hat{y}_i)^2$ is minimized.

For example, if we were just to guess that $\beta = 1.0$ and $\alpha = 0$, then our subset of $\hat{y}_i$ would be:

$$\hat{y}_1 = \alpha + \beta x_1 = 0 + 1(1.0\%) = 1.0\%$$
$$\hat{y}_2 = \alpha + \beta x_2 = 0 + 1(2.0\%) = 2.0\%$$
$$\hat{y}_3 = \alpha + \beta x_3 = 0 + 1(0.5\%) = 0.5\%$$

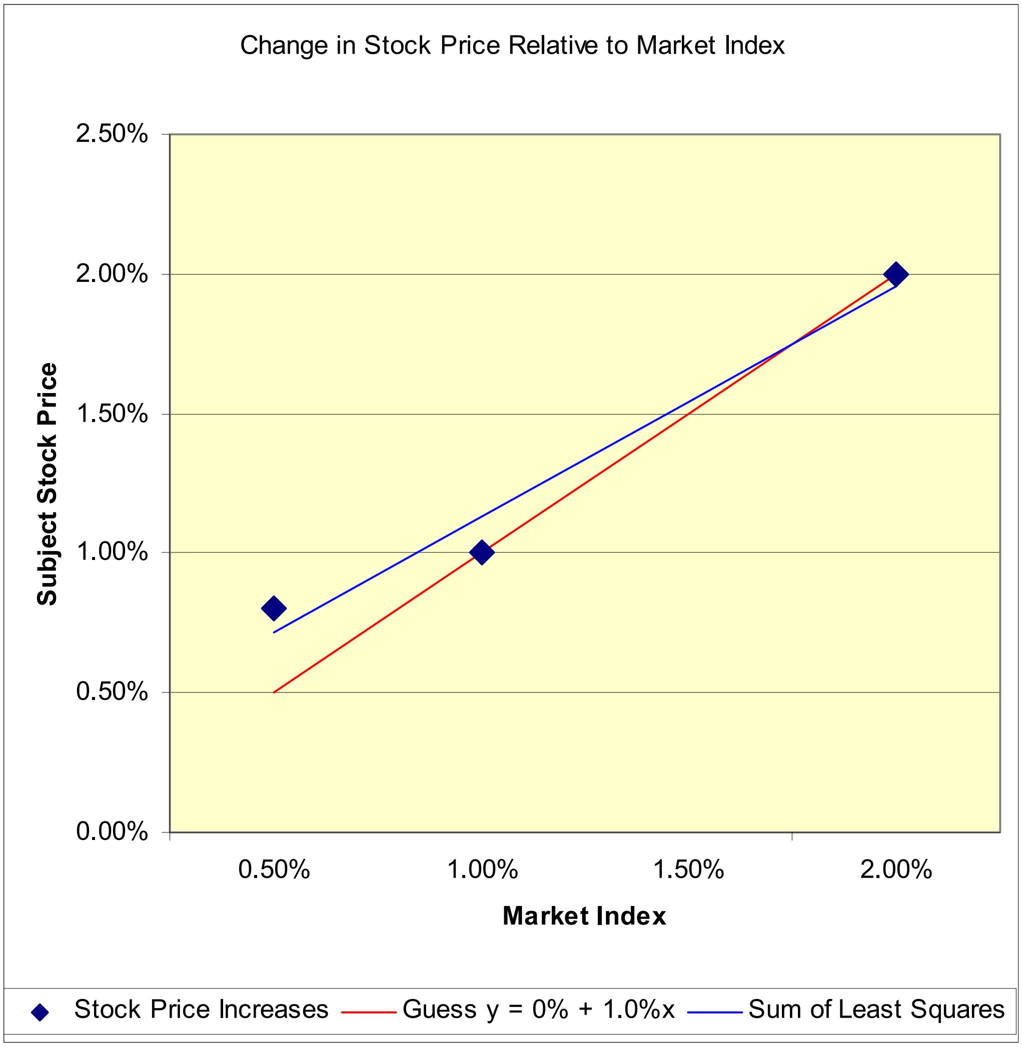
And, $\Sigma (y_i - \hat{y}_i)^2 = (y_1 - \hat{y}_1)^2 + (y_2 - \hat{y}_2)^2 + (y_3 - \hat{y}_3)^2$
$= (1.0\% - 1.0\%)^2 + (2.0\% - 2.0\%)^2 + (0.8\% - 0.5\%)^2$
$= 0.09\%$

And this guess may lead to a reasonably accurate prediction of the best fitted regression line – after all, 9/100ths of one percent is a very small number. However, any basic statistical program will tell us that variables that lead to the absolute least squares is $\beta = 0.828571$ and $\alpha = 0.003$, now:

$$\hat{y}_1 = \alpha + \beta x_1 = 0.003 + 0.828571(1.0\%) = 1.1286\%$$
$$\hat{y}_2 = \alpha + \beta x_2 = 0.003 + 0.828571(2.0\%) = 1.9571\%$$
$$\hat{y}_3 = \alpha + \beta x_3 = 0.003 + 0.828571(0.5\%) = 0.7143\%$$

And, $\Sigma\ (y_i - \hat{y}_i)^2$ [with $(y_i - \hat{y}_i)^2$ rounded to the 4th decimal] = 0.0561%, which is a measurably smaller number than 0.09%

Graphically, we can see that the Least Squares line of $\hat{y}_i = 0.003 + 0.828571x_i$ (blue line) represents a closer fit to the three actual data points that does our intuitive guess of $\hat{y}_i = 0.0 + 1.0x_i$ (red line).



The vertical distance (y_i - $\hat{y}_i$) is also known as the **residual** (also known as the error of estimation). It represents that portion of the actual data that the regression was not able to explain. In the case where ALL the observed data points lay exactly on the regressed line, the sum of the residuals would be zero. The regression, in that case, would have explained 100% of the stock price movement with respect to a change in the Market Index, and, as we shall subsequently discuss, the R-Squared in that instance would be 1.0. The residual, at x = 0.50% above, where the actual y_3 data is 0.8% less the regressed $\hat{y}_3$ of 0.7143% is 0.0857%. Intuitively, it should be apparent that, when all other things are equal, a regression with a lower absolute sum of total residuals is a "better fit" than one, in every other way identical, but having a greater absolute sum of residuals.

In the “Guess” regression above, the absolute sum of the residuals is:

ABS(1.0% - 1.0%) + ABS(2.0% - 2.0%) + ABS(0.8% - 0.5%) = **0.3%**

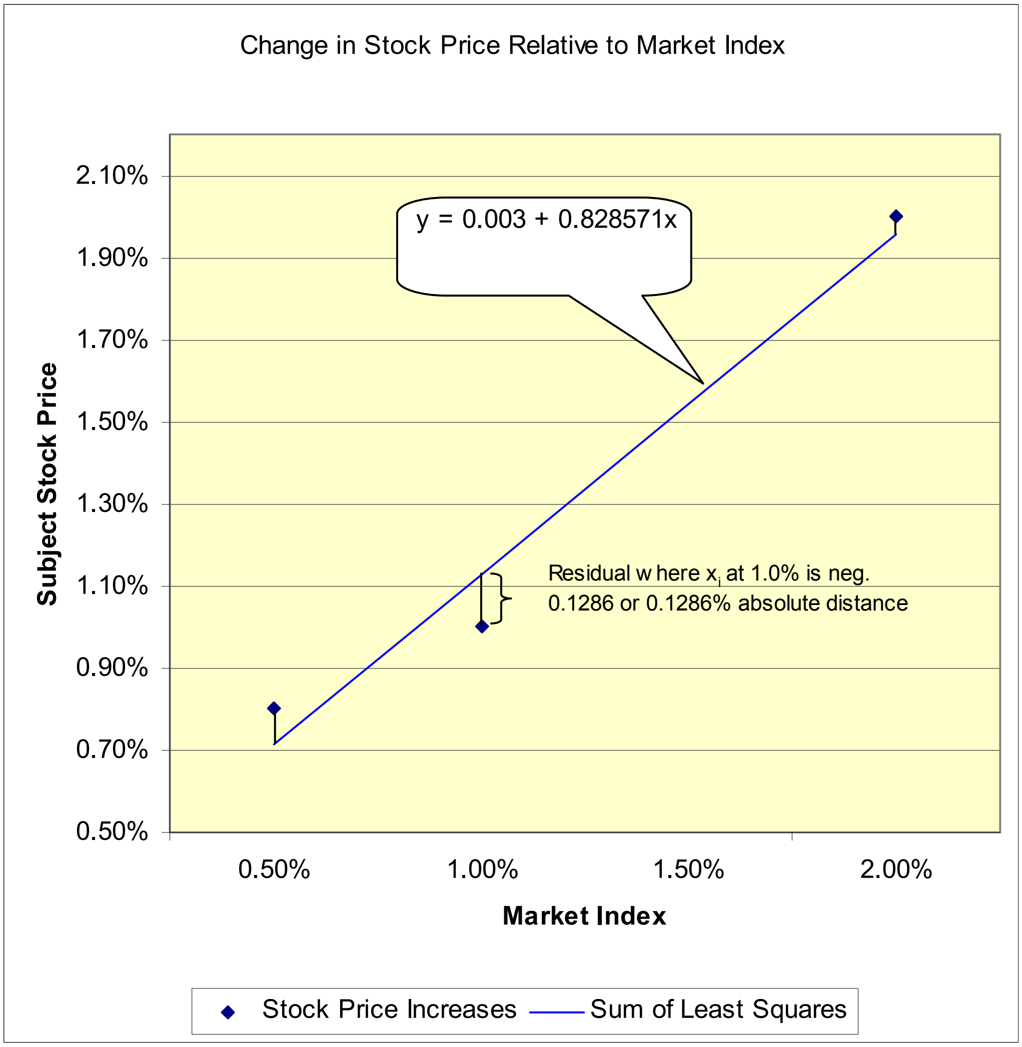
Whereas, in the Least Squares Regression it is:

ABS(1.0% - 1.1286%) + ABS(2.0% - 1.9571%) + ABS(0.8% - 0.7143%) = **0.2572%**

And,

0.2572% < 0.3%, therefore the Least Squares regression must be the better fit[7]. Note that this occurs even in spite of the fact that, in the “Guess” regression, two of the three data points have residuals that are precisely zero (i.e. are exactly on the regressed red line).

[7] The mathematically astute will recognize that the nominal (non-absolute) total of the residuals will equal zero. That is, the negative residual total will always equal the sum of the sum of the positive residuals. This is a required property of the Least Squares method. Conceptually, however, it is easier to imagine that we are attempting to minimize the total absolute distance between the actual observed data points and corresponding point on the fitted line (i.e. the errors of estimation). When the sum of the absolute errors of estimation has been set to their lowest possible value, this line represents the least squares line of best fit.



STANDARD ERROR OF ESTIMATION: Another important indication of the quality of the regressed beta line is the standard error of estimation. The standard error of estimation is the sum of the squared residuals, divided by the number of degrees of freedom in the sample, or:

$$\Sigma (y_i - \hat{y}_i)^2 / (n - 2)$$

Where (n – 2) represents the degrees of freedom. A theoretical explanation of the degrees of freedom will not be attempted here. However, the “n” represents the number of observations in the sample. Therefore, if the regression has been made from the customary 60 months of stock data, n = 60 and the degrees of freedom would be 58.

Intuitively one should recognize the Standard Error of Estimation as a measure of variability around the regression line [hence, the (y_i - $\hat{y}_i$) residual inclusion and the divisor of n – 2 attempts to find the average amount of variability per sample observation].

Similar to finding the absolute sum of residuals, a *lower* Standard Error Estimation, all other things being equal, indicates a better quality of regression.

In the preceding example, we already know that, for the Guessed regression line $\Sigma (y_i - \hat{y}_i)^2 = 0.09\%$ and that there were only three observations in the sample[8], therefore n – 2 = 1, so the Standard Error of Estimation = 0.09%. Conversely, in the Least Squares Regression, $\Sigma (y_i - \hat{y}_i)^2 = 0.0561\%$ and this, divided by 1 also becomes the Standard Error.

R-SQUARED: The formula representing the R-Squared Statistic is:

$$R^2 = (\hat{y}_i - \bar{y})^2 / (y_i - \bar{y})^2$$

Where:

$\hat{y}_i$ still represents a y data point on the regressed line calculated by $\alpha + \beta x_i$
$\bar{y}$ is the mean value of all the actual y observations in the sample
and
y_i is, of course, the actual y value at that observation

It is extremely instructive to visualize the two main components of this ratio, namely:

$(\hat{y}_i - \bar{y})$ and $(y_i - \bar{y})$

The first term represents the physical distance from the fitted regression line to the line that is the average of all the y observations. The second term is the physical distance from the single y_i data point to the line that is the average of all the y observations. Graphically, highlighting just one data observation, this would be:

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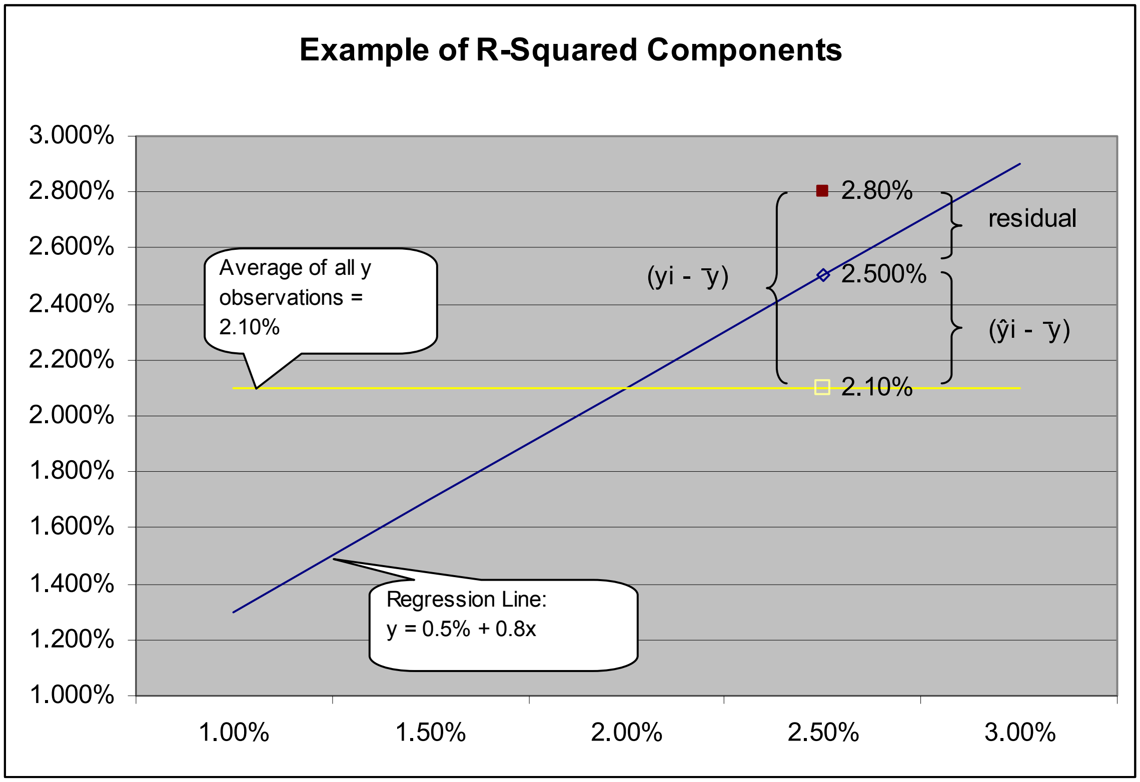
[8] A sample of only three observations would never be found to be statistically viable, but it has been used here just for ease of presentation.

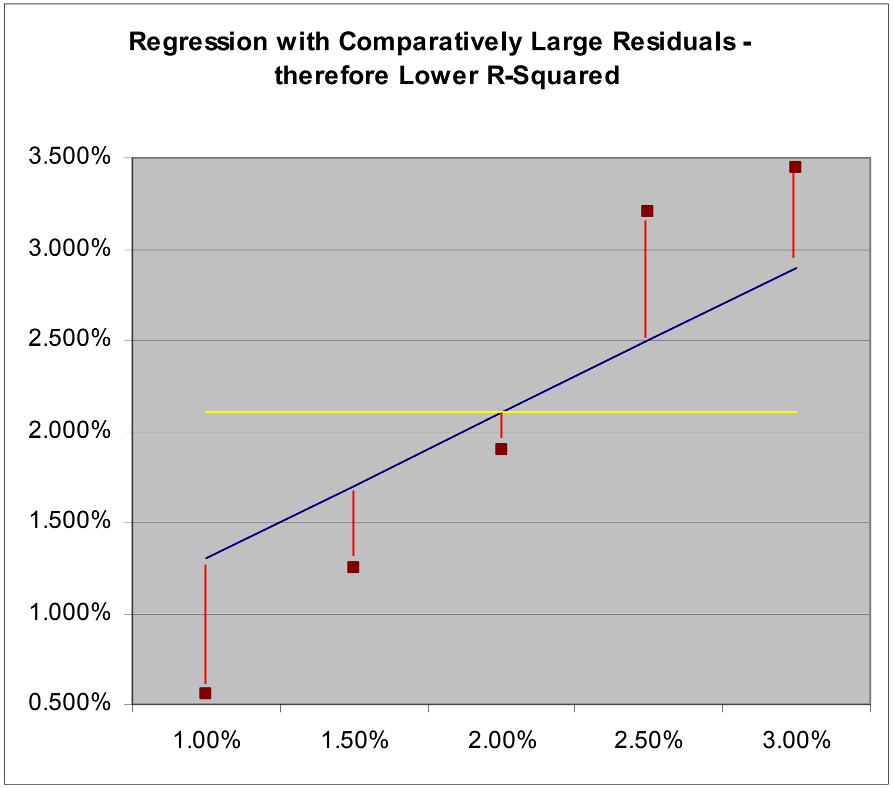


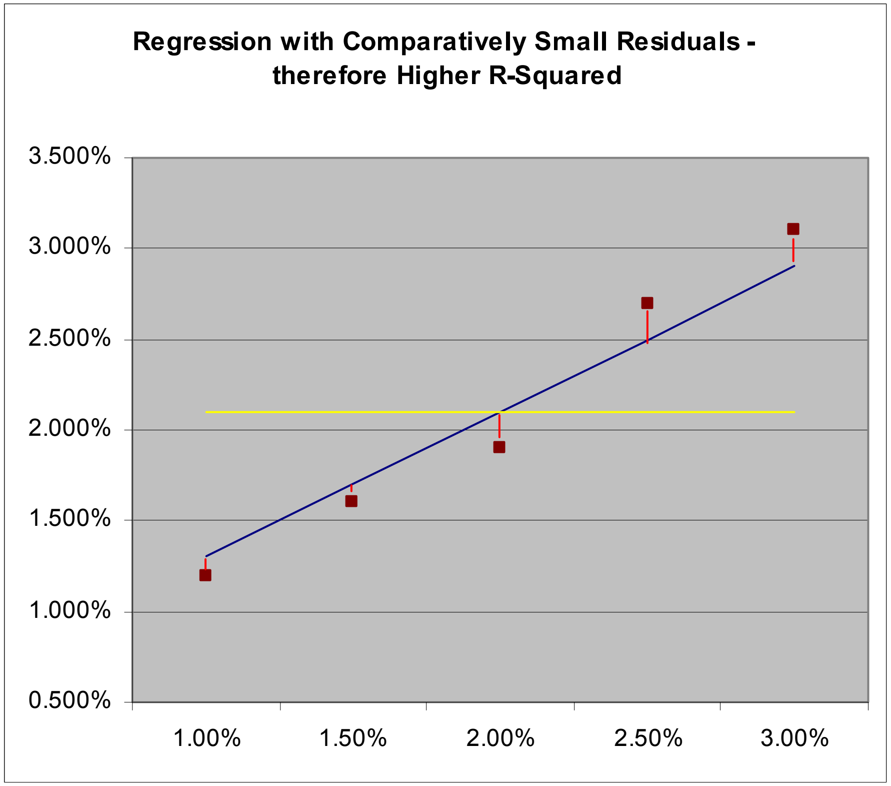
Notice that the $\bar{y}$ is just a common starting point for both terms. $(\hat{y}_i - \bar{y})$ is the physical distance that the regression is able to explain (i.e. from the data average to the placement of the fitted line). The total distance is $(y_i - \bar{y})$. Therefore, the ratio: $(\hat{y}_i - \bar{y}) / (y_i - \bar{y})$ [9] represents the total proportion of the movement around y that the regression was able to explain which is why the R-Squared Statistic is also known as a "goodness of fit" measure. That proportion of the distance between average y and y_i that the regression was not able to explain is the residual or error of the estimation. Note that the residual is equal to $(y_i - \bar{y}) - (\hat{y}_i - \bar{y})$. Numerically, in the above graphic, the residual is equal to (2.80% - 2.10%) – (2.50% - 2.10%) = 0.30% = $(y_i - \hat{y}_i)$

Since the residual represents the "unexplained" proportion of the regression – the part that goes beyond the territory of the fitted line – it should be intuitively obvious that the larger the total absolute distance of the residuals are, relative to the distance of the

[9] Squaring both terms just eliminates the possibility of negative numbers, so that the sum of the terms will always be additive.

regression line back to the average y line, the lower the quality of the regression. In the following two graphs, having exactly the same fitted regression line and same average y line, for example, the one on the left has a much higher total absolute residual distance (represented by the red bars). The one on the right has a much smaller total absolute residual distance and would, accordingly have a much higher r-squared value.


Regression with Comparatively Large Residuals - therefore Lower R-Squared
3.500%
3.000%
2.500%
2.000%
1.500%
1.000%
0.500%
1.00%
1.50%
2.00%
2.50%
3.00%


Regression with Comparatively Small Residuals - therefore Higher R-Squared
3.500%
3.000%
2.500%
2.000%
1.500%
1.000%
0.500%
1.00%
1.50%
2.00%
2.50%
3.00%