VALUATION

Modigliani-Miller Propositions I & II as Applied to Business Valuation

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Imagine a non-taxable firm that has a 10 percent unlevered cost of equity and has a 5 percent market cost of debt. This firm has a 50:50 debt/equity structure. Is it possible for such a firm to have a WACC (weighted average cost of capital) of:

> 10% x 50% + 5% x 50% = 7.5% ?

The answer is no, and our first clue should be the erroneous inclusion of an unlevered (i.e., calculated assuming the absence of debt) cost of equity rate in a WACC where debt exists. Such an observation begs the question, "What part of finance theory speaks to the distinction between levered and unlevered rates, and how does this impact the everyday valuation of private firms?"

The short answer is that a widely accepted tenet of finance theory tells us that such a firm, in the absence of any tax shield effects, could never have a WACC lower than 10 percent (the unlevered cost of equity). In fact, 10 percent is the lower bound on WACC regardless of how much debt the firm takes on or what the market rate of borrowing is. If your first inclination was that the blended WACC of 7.5 percent was correct, then this review of Modigliani-Miller theory is for you.

In 1958 Franco Modigliani and Merton Miller published "The Cost of Capital, Corporation Finance and the Theory of Investment," which they followed up in 1963 with "Corporate Income Taxes and the Cost of Capital: A Correction." Both papers have met with considerable opposition over the years, and parts of the theory are still hotly debated by finance academics today. But the core of the theory has largely been accepted by financial theorists the world over, and the proposals contained in those initial works have become mainstays of modern finance. Indeed, the theorems proposed have become so ingrained in the theory of finance that we typically refer to them simply as MM I & MM II (Modigliani-Miller Proposition I and II).

The purpose of this article is to serve as a brief and very informal[1] review of MM I & II, and why they are still so important to the understanding of business valuation. In fact, I suggest that MM I is even more applicable to closely held private equity than to the public corporate structures it was initially meant to explain. This is because control tends to be highly concentrated in private firms and, typically, the capital structures and covenants attached to the various securities are usually quite simple. In any case, MM I & II are extremely relevant to the valuation of small private firms, and this article is meant to serve as a high-level refresher on these important theorems.

MM I

When Modigliani and Miller initially described the parameters of Proposition I in 1958, they did so using variables. My example is going to use absolute numbers because I think that it is easier for those of us who evolved out of the accounting world to conceptualize. MM began Proposition I by asking us to consider two identical firms (Companies A and B, to which we shall refer throughout the article.); same risk profile, same prospects, same products, industry and markets. Each is expected to earn a net pre-interest[2] annual income of $100,000. Initially we will presume both are exempt from income taxation. In fact, the *only* difference between the two firms is that Company A is financed entirely by equity, whereas Company B had a mixture of debt and equity (see Table 1, page 8).

Both firms have recently undergone an extensive appraisal process, where it was determined that the enterprise value of Co. A was $1 million and Co. B was $1.1 million. For simplicity, the market value of debt is assumed to be the same as the

1. I make no pretense of treating this subject academically, nor is it necessary to do so. The beauty of the MM propositions is that they are largely based on common sense and the absence of arbitrage potential.
2. For simplicity, we will ignore the tax shield generated by the interest expenses until a later example.

TABLE 1: COMPARISON OF COMPANIES A & B CAPITAL STRUCTURE

	($000)	
	CO. A	CO. B
Expected perpetual annual income: zero-taxes and before interest expenses	100	100
Appraised value of shares	1,000	600
Market value of debt		500
Enterprise value	1,000	1,100
Coupon rate, interest paid annually		5%

face value[3] and annual income is assumed to be equal to cash flows.

Assume that $100,000 of shares of Co. B are owned by Investor X and, like all rational, risk-averse investors, he seeks to maximize his returns relative to the risk he is willing to bear. Currently the annual interest payment on the debt is $25,000 ($500k x 5%) and this leaves $75,000 x (100k/600k [being Investor X's proportional share of post-interest income]) = $12,500 in perpetuity for Investor X to receive in dividends each year. This equates to a 12.5 percent annual return on his $100,000 investment. In contrast, it is obvious that the shareholders of Co. A are receiving only a 10 percent annual return on their $1 million in equity—so, intuitively, it makes sense that Investor X would wish to continue holding what appears to be the superior equity of Co. B.

MM I argues that the hypothesized scenario could never reflect actual market equilibrium—specifically that the appraised market value of Co. B must be incorrect. The MM reasoning is that an arbitrage opportunity exists such that Investor X can increase his overall returns without incurring any additional risk. They propose that, in order to maximize his returns, Investor X will sell his $100,000 of Co. B shares and invest the $100,000 of proceeds into Co. A shares (purchasing these from existing Co. A shareholders). Investor X will not stop there. He will go to the bank and borrow (100k/600k) x $500k in order to purchase an additional $83,333[4] of Co. A equity. In order to complete this transaction as a riskless arbitrage, MM I imposes two necessary conditions:

- That Investor X can borrow the $83,333 at the same 5 percent rate that Co. B enjoyed (and, considering that Investor X could be another corporate entity with exactly the same creditworthiness as Co. B, this is entirely believable).
- That the bank would accept the Co. A shares as the only collateral required on the $83,333 debt (and this condition is also reasonable given that Investor X will be offering up a total $183,333 of Co. A shares as security on $83,333 of debt).

After Investor X has effectively switched horses, his or her annual expected net return after making the interest payment on the borrowed $83,333 would be $100k x (183.3k/1,000k) – ($83.3k x 5%) = $14,100 on a $100,000 investment, or a 14.1 percent return. The annual yield has increased by 160 basis points over the 12.5 percent he or she was receiving with Co. B. Moreover, the overall risk exposure has not changed one iota. Previously, as a Co. B shareholder, the worst case scenario would be if the firm failed and the shares were deemed worthless. In this case, Investor X would lose $100,000 in principal, but the limited liability nature of the equity position would protect Investor X from any additional losses. The liquidation value of the firm (if any) would effectively pass to the Co. B bondholders. Now, as a Co. A shareholder, the worst case scenario would still occur should the firm fail and the shares are deemed worthless—but Investor X bears no additional liability on the $83,333 of debt owed to the bank. The bank had agreed to be entirely collateralized by the value of the Co. A shares. Investor X would still lose $100,000 in principal, but escape any liability on the borrowed funds. 18.3 percent of any residual liquidation value in the firm would pass to the bank when it took possession of its collateral.[5]

3. Or, if you wish, imagine the coupon rate is a fixed 5 percent but payable on the market value, rather than on the face value of the debt. This eliminates the need to consider bond discounts and premiums.

4. Investor X's purpose here is to transfer his proportional interest in Co. B to Co. A. Further, because Co. A does not hold any debt, Investor X knows he can "self-construct" that same debt-to-equity ratio that B had by borrowing the money on his own account and then using those funds to purchase more A shares.

5. Note that the bank is, by proxy, taking on a very similar credit risk in Co. A as the bondholders were willing to take with Co. B. Investor X is counting on this and has constructed his new levered Co. A stock portfolio on this basis.

MM I argues that perfect capital markets will not allow riskless arbitrage opportunities to exist.[6] Other Co. B shareholders would quickly see the advantage of selling B shares, self-leveraging, and buying A. Soon the price of B would be bid down until the enterprise value of B was identical with A. The example works in the other direction as well (when the 100 percent equity firm has a higher appraised value than the mixed-capital firm, in which case the Co. A shareholders sell), but I will forgo that in the interests of brevity.

While it may initially seem counter-intuitive, the bigger picture makes sense and appeals to our belief that the markets allocate capital to those that use it most efficiently. In this example, given that both firms had *precisely* the same risk profile and were expected to earn exactly the same tax-exempt income into perpetuity, then Co. A is obviously the more efficient firm. Co. A only requires $1 million in capital to generate $100,000 income annually; whereas Co. B requires $1.1 million to generate exactly the same amount incurring the same risks. Put another way, if you were an entrepreneur looking to buy all the outstanding equity and debt of one or the other of these two businesses, Co. A would cost $1 million and generate an expected 10 percent return while Co. B would cost $1.1 million and only generate a 9.1 percent return to total capital invested. Knowing that the inherent risks are identical, Co. A is the better investment.

Capital structure does not matter (within a relevant range)—that is the primary theme of MM I: "The market value of any firm is independent of its capital structure."[7] Nowhere is this truism more evident than in the valuation of small, closely held businesses. This is because control often resides with just one individual (or a family all with the same vested interests) and capital structure is usually determined by tax efficiencies.

Referring to Co. A again as an example, a different small business investor, Investor Y, could purchase all the shares of the firm and immediately have the firm repurchase, say, 50 percent of the outstanding shares. He could then write up a debenture that lends the $500,000 he got on the share repurchase back to the firm at a rate that appropriately reflects the correct risk-adjusted rate on similar bonds. Now Co. A would be 50 percent debt-financed and 50 percent equity-financed. At the end of every year, Investor Y now has to write himself two checks, one for the interest payment and another dividend check. The total of these two checks would amount to $100,000—exactly the same as when the firm was 100 percent equity-financed. But how else has this refinancing changed the firm? Its prospects, risk exposure, human capital, customer and supplier bases, sensitivity to unforeseen shocks, intangibles, and macroeconomic environment all remain completely unaltered by this reclassification of equity to debt. The fact is, none of the determinates of value have changed, and, if 10 percent was the appropriate discount rate to be applied when the firm was 100 percent equity-financed, then, notwithstanding the interest tax shield impacts, the firm will still have this same $1 million value regardless of how the single stakeholder rearranges the capital structure.

And, viewed from the sole stakeholder's perspective, excluding the tax shield advantages of debt, why would he care how his $1 million was arranged?[8] When the firm is 100 percent equity-financed, Investor Y is entitled to an annual $100,000 dividend check; when the firm is financed by 50 percent debt, he is entitled to two annual checks that total $100,000. If the firm starts doing better than expected, the equity portion of his portfolio will accrete in value. Conversely, if the firm begins enduring hard times and it seems as if the bonds are going to default on their interest payment, it is hardly likely that the sole stakeholder is going to sue his own company. The reality is that when there is only one stakeholder (or one close-knit group that have homogeneous interests), his risk exposure runs parallel with the underlying economics of the firm itself—the capital structure is irrelevant.

What might happen if the single stakeholder sold the $500,000 of Co. A bonds to some unrelated third-party investor? How does this impact the value of Co. A? Well, it doesn't. That annual interest check is going to have a different payee. None of the underlying economic determinants of value for the firm have been altered by the fact that the bonds are now in different hands. However, in the event of a potential default on the bonds, the fact that the bondholder and the shareholder are now two unrelated parties (often with opposing interests) will alter the actions the bondholder can be expected to take in protecting his interests. And these actions will, in turn, inflict transaction

6. Recall that the first MM publication predates Black Scholes by approximately 15 years when it then became the accepted norm to price options by a no-arbitrage methodology.

7. Franco Modigliani and Merton H. Miller, "The Cost of Capital, Corporation Finance, and the Theory of Investment," *American Economic Review,* 1958 (48:3, page 268).

8. In the event of bankruptcy, he may prefer to be a secured creditor rather than holding 100 percent equity and no secured debt. MM I is limited in this regard and the costs of distress are discussed subsequently.

costs upon the firm that will alter firm value. Note also that Investor Y is now no longer indifferent to the allocation of earnings. If Co. A's marginal tax rate is 40 percent, then for every dollar paid to the bondholder for interest there is $0.60 less to be paid to Investor Y in dividends. Further, Investor Y, as sole shareholder and manager of the firm, may come to realize that should the firm fail, he now has only $500,000 at risk rather than the full $1 million. Conversely, any additional profits earned above the fixed annual interest payments accrue directly to his benefit. This realization may motivate Investor Y to take on increased risk in the daily management of the firm, and these increased agency costs will, in turn, cause firm value to change inversely in proportion to the increased risk.

CRITICISMS OF MM I

The prevailing criticism of MM I is that it is founded upon a static model of default-free debt. The presumption is that the issuing firm is financially sound and there is no more financial leverage risk at a 7:3 debt/equity ratio than there would be at 3:7. It does not consider what would happen to the cost of debt should the debt coverage ratio decline and the firm draws nearer to default or bankruptcy. Nor does MM I allow for a change in agency costs or the transaction costs incurred in less-than-perfect markets (such as the difference in issuance costs of equity versus dqebt). This does not invalidate the usefulness of MM I, but does relegate its applicability to a relevant range. MM I is a fair-weather cost of capital model, and it cannot be expected to produce meaningful results for those firms where the potential for financial distress ranks high.

MM II IMPLICATIONS FOR WACC

The primary concept of MM II is that the cost of levered equity continues to rise as proportionally more debt is added, such that the overall company cost of capital does not change. In other words, the benefits of using more debt in the capital structure are exactly offset by the increase in the cost of equity (we are still ignoring the effects of the tax shield here).
MM II can be symbolized as:

$$k_L = k_U + (k_U - r_d)(D/E_L) \quad (1)$$

Where:

k_L = levered cost of equity
k_U = unlevered (i.e., 100% equity) cost of equity
r_d = cost of debt (market rate)
D = market value of debt
E_L = market value of equity (given that the firm is levered)

In Table 2 we apply this formula to Company A where unlevered cost of equity is 10 percent, and assuming a wide array of possible debt-to-equity ratios and varying cost-of-debt rates.

TABLE 2: COMPANY A'S WACC, ACCORDING TO MM II (TAX-EXEMPT)

UNLEVERED COST OF EQUITY (%) K_U	MARKET RATE OF DEBT (%) k_d	DEBT : EQUITY RATIO D/E_L	LEVERED COST OF EQUITY PER MM II (%) K_L	KL[EL/(D + EL)] + KD [D/(D + EL)] = WACC (%)
10.0	5.0	0 : 10	10.0	10.0
10.0	5.5	1 : 9	10.5	10.0
10.0	6.0	2 : 8	11.0	10.0
10.0	6.5	3 : 7	11.5	10.0
10.0	7.0	4 : 6	12.0	10.0
10.0	7.5	5 : 5	12.5	10.0
10.0	8.0	6 : 4	13.0	10.0

The key thing to note with respect to Table 2 is that neither the market rate of debt nor the proportionate amount of debt within the capital structure has any impact upon WACC. It remains equal to the unlevered cost of equity at 10 percent.

Returning to our Co. B example above, where k_U = 10%, r_d = 5%, D = $500k and E_L = $600k:

$$k_L = 10\% + (10\% - 5\%)(500k/600k) = 14.16\%$$

MM II [formula (1)] tells us that the Co. B levered cost of equity should be 14.16 percent. Applying this to the well-known WACC formula (still ignoring any potential tax shield impact):

$$WACC = 14.16\%(600k/1{,}100k) + 5\%(500k/1{,}100k) = 10\%$$

The Co. B WACC of 10 percent turns out to exactly equal the unlevered cost of equity. Again, the fundamental premise of MM I & II is that capital structure does not matter. Therefore, it is by design that the no-tax MM II model always produces a WACC that equals k_U. Indeed, on a tax-exempt basis, the MM II formula will give us the correct WACC even when the components of debt and equity have been mispriced as they have in Co. B's case.

This fact emphasizes what the MM I no-arbitrage argument has already brought to light: The Co. B fair market value could not possibly be $1.1 million as initially appraised. Given that the appropriate WACC is 10 percent and the expected annual cash inflow is $100,000 in perpetuity, the enterprise value of Co. B must be $1 million ($100k/10%), not $1.1 million.

INCORPORATING TAXES

MM did recognize that there was an exception to the "capital structure does not matter" premise when the tax treatment between interest expense and dividends exists (as it does in most countries). The tax deductibility of interest expenses[9] creates a tax shield that is not enjoyed by dividend payments. The value of the firm goes up by the present value of the interest tax shield because the government is effectively subsidizing the firm by allowing interest payments to be deducted from taxable income but excluding dividends from this same treatment. Simply stated, interest expense reduces the amount of cash taxes payable,[10] and the present value of these future savings directly increases firm value.

MM II can be altered to incorporate the benefits of the tax shield by adding (1 – T) to the formula:

$$k_L = k_U + (k_U - r_d)(1 - T)(D/E_L) \qquad (2)$$

9. Subject to various limitations, jurisdictional variations, and thin capitalization rules.
10. The unstated implication here is that the firm is currently or soon will be "cash taxable" and will be able to apply 100 percent of the incurred interest expense deductions against taxable income.

where T is the applicable marginal corporate tax rate for that firm. Assuming Co. B's marginal tax rate is 40 percent:

$$k_L = 10\% + (10\% - 5\%)(1 - 40\%)(D/E_L)$$

Strictly speaking, we no longer know what the correct market values for the D/E_L ratio is (500k/600k was wrong in the first place and certainly does not incorporate the benefits of the tax shield anyway). This matters in the tax-adjusted MM II formula because it impacts the amount of value attributed to the tax shield.

We can deduce the missing information by recognizing that the only difference MM would allow between a levered and an unlevered firm is the present value of the tax shield. Assuming that the debt rate is the correct risk rate to discount the future tax savings (and this was the primary conclusion of the 1963 MM "correction"), then the incorporation of a $25,000 annual interest expense will generate an annual $10,000 ($25k x 40%) in tax savings. A perpetuity of $10,000 discounted at 5 percent has a present value of $200,000 ($10k/5%).

Therefore, tax-adjusted Co. B must have a fair market value of the 100 percent equity firm (such as Co. A) plus the $200,000 tax shield, or $1.2 million. Further, if we can rely upon the fact that true fair market value for the debt is $500,000 (perhaps because an arm's-length, third-party purchaser just paid that much for the bonds), then the fair market value of the levered, tax-adjusted equity must be $700,000 and:

$$k_L = 10\% + (10\% - 5\%)(1 - 40\%)(500k/700k) = 12.14\%$$

The WACC tax-adjusted formula becomes:

$$WACC = k_L(E_L/(D + E_L) + r_d(1 - T)(D/(D + E_L)) \qquad (3)$$

$$WACC = 12.14\%(700k/1{,}200k) + 5\%(1\text{-}40\%)(500k/1{,}200k) = 8.3333\%$$

And, not surprisingly, a perpetuity of $100,000 discounted at 8.3333 percent does confirm the correct enterprise value of $1.2 million. Note that the present value of the tax shield always accrues to the benefit of the shareholders. In this case, the value of Co. B without the tax shield would have been $1 million (same as unlevered Co. A), and therefore the equity would have had a fair market value of $500,000; but this is increased to $700,000 as a result of the tax shield.

Rearranging formulas (2) and (3) we can isolate the net impact that the introduction of the tax shield will have on the unlevered WACC (i.e., 100 percent equity WACC; for convenience I will refer to the 100 percent equity structure as the unlevered WACC rate). That formula is:

$$\Delta WACC, \text{ Unlevered to Levered} = -TDk_U/(D + E_L) \qquad (4)$$

In the Co. B example:

$$\Delta WACC = (-40\% \times 500k \times 10\%)/(500k + 700k) = -1.6666\%$$

Which, of course, represents the decline in the unlevered 10 percent WACC to the 8.3333 percent tax-adjusted levered WACC.

CONCLUSIONS

Regarding the applicability of MM I & II in the valuation of private equity:

1. On a no-tax basis, changing the capital structure will not result in any change in the WACC. Regardless of the mix of debt-to-equity, the WACC cannot descend lower than the unlevered cost of equity.
2. Considering the present value of the interest expense tax shield, the levered WACC will be lower than the unlevered cost of equity rate—but never by more than $-TDk_U/(D + E_L)$. In other words, the difference in fair market value between the 100 percent equity-financed firm and the levered firm will never be more than the present value of the tax shield generated by the future stream of interest expenses.

The reason why MM I & II play such an important role in the valuation of closely held private equity is because almost every valuation assignment first requires the identification of the unlevered cost of equity. Often, it then becomes necessary to recommend the adoption of an "optimal capital structure," perhaps to reflect the industry standard D/E_L ratio that is customary in the subject's peer group. The important thing to realize, however, is that MM I & II sets an upper bound on how much the value of the firm will increase as a result of the integration of debt into the capital mix. That increase is restricted to, at most, the present value of the tax shield. Given a 10 percent cost of unlevered equity, and similar input variables as described above, then MM I & II say that the lower bound on the achievable rate of levered WACC is 8.3 percent. If you have somehow arrived at a 6.0 percent levered WACC, then you are traveling through another dimension. A

dimension not only of sight and sound but of mind…however, not a dimension in which modern financial theory will support you. In the past 50 years, since the introduction of the MM Propositions, there has been no creditable theory advanced, that I am aware of, which asserts that the increase of debt in the capital structure results in a lower overall cost of capital (save for the inevitable benefit of the tax shield, of course).

In fact, since the introduction of the MM theories, a good deal of empirical evidence and scads of academic papers have argued in the other direction—that the introduction of debt into the capital structure eventually, if not immediately, leads to increases in the WACC. When debt is proportionately low compared to equity, and the debt coverage ratio is high, the value of the firm will initially increase as a result of the present value of the tax shield. However, as more debt is added and potential for default increases, investors begin pricing in the costs of financial reorganization or bankruptcy; debt holders begin demanding higher coupon rates to compensate them for the increasing risk of default and this, in turn, causes the equity investors to feel squeezed and demand an increasing levered equity rate in compensation for the fact that, once the bondholders get their bigger piece of the pie, there now might not be enough left over to pay dividends.

For all these reasons, MM I & II serve as a good double-check on the lower bound of levered WACC for going-concern firms. MM I & II are not well suited for distressed firms—and never were intended for that purpose. For distressed firms, it is more likely that the addition of debt will increase WACC. VE

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