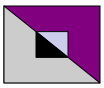

Accession Capital Corp

ESTIMATING FUTURE GROWTH IN APPRECIATING REAL PROPERTY

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EXECUTIVE SUMMARY

- Future asset appreciation expectations must be priced into existing market assessment of current value
- Purpose of paper is to consider one possible method of quantifying those expectations using the Gordon Growth Model
- Future estimations of asset value should have an explicable growth profile that are consistent with underlying economics and risk/return expectations
- In the case at hand, the appraiser has failed to support his opinion of future value

INTRODUCTION

The case deals with that concept that we all intuitively must recognize when considering the current value of a generally appreciating asset (specifically land) but seldom actually attempt to quantify. That concept is that, at any given point in time, the market value of a tract of land *must* include all future expectations of price appreciation. If it did not, the property would be undervalued and an arbitrage opportunity would be available. Unlike depreciating or depleting assets, the value of most land – certainly that situated near large urban centers and free from environmental contamination or other future liabilities – will continue to appreciate in value.

The purpose of this paper is to consider some simplistic means of quantifying those future appreciation expectations and speculate upon how those expectations impacted this one transaction.

FACTS

The property appraiser had a considerable challenge in this project. The task was to determine an opinion of the Present Value (PV) of a 30 year lease extension that was already under lease for the next 45 years. Not only were all the usual complexities of attempting to quantify the value of a long-term legal commitment present in this

assignment, that 30 year commitment was not even to come into effect for another 45 years hence, added even more uncertainty to the outcome. The present value of those rights were to be determined in current day dollars so that the 30 year extension would be locked-in and entirely paid for in one lump-sum amount 45 years in advance of the start of the lease term. In brief:

- $48.482 million was the current estimate of the market value purchase price of the land (an assessment well-supported by recent comparative land transactions in the area)
- 6.0% discount rate was presented as the appropriate risk-adjusted return application to the transaction (well supported by references to other recent real property transactions of a similar nature – both as leases and outright purchases). 6.0% appeared to be approximately what the market was demanding on similar real estate transactions.
- Existing lease term set to expire in 45 years
- Remainder of existing 45 year lease term had been entirely prepaid many years prior when market conditions were significantly different
- Desired lease extension was an additional 30 years

APPRAISER'S METHODOLOGY

The appraiser began quite rationally and, having documented and supported strong market comparatives showing that the outright purchase price for the land, had it been available for sale at that time, would have come in just over $48 million. Current market expectations of return on similar transactions were at an approximate 6.0%. So the appraiser argued that, if the land owner was able to lease the land for a full 75-year fixed term (from current day until the end of the projected 30 year extension), his expectations would be to receive an annual $2.909[1] million lease payment ($48.482 x 6.0%) on the

[1] For ease of presentation, all the lease payments presented herein will be IN ARREARS rather than the IN ADVANCE cash flows that would normally be expected with any form of lease payment. The financial difference this assumption makes is not significant and has no bearing on the salient points of the case.

property. Such an outcome would be in accord with current market conditions. He then correctly pointed out that the Present Value (PV) of such a cash flow would equate to $47.869 million for the entire hypothetical 75 year lease[2].

So far so good. Of course, the assignment was to find out the PV of a 30 year lease would be starting 45 years hence. Here, without a great deal of supportive detail the appraiser concludes that the one-time, lump-sum PV of the final 30 years would amount to **$14.414 million**. If we accept the strength of the appraiser's argument with respect to the current purchase price and the risk-adjusted discount rate - and that the landowner would be fairly compensating for accepting 2.909 million per year for the next seventy-five years, then we must absolutely acknowledge that it would be just as equally fair to arrange to pay the landowner via two leases, one 45 year term and then a subsequent 30 year each paying $2.909 per year. Whether the landowner holds two identical contracts for a 45 and then 30 year lease vs. a single 75 year lease is immaterial. In both cases the landowner's net cash flows would be identical as would his legal entitlements.

A) PV of Single 75 Year Lease with $2.909 million in Annual Lease Payment (Arrears):

$$\$2.909/.06\ (1 - 1.06^{-75}) = \mathbf{\$47.870}$$

B) PV of a 45 Year Lease with 2.909 million in Annual Lease Payment (Arrears):

$$\$2.909/.06\ (1 - 1.06^{-45}) = \mathbf{\$44.961}$$

C) PV of a 30 Year Lease with 2.909 million in Annual Lease Payment (Arrears) – Not Scheduled to Begin until 45 Years Hence:

$$(\$2.909/.06\ (1 - 1.06^{-30})) \times 1.06^{-45} = \mathbf{\$2.909}$$

[2] $2.90892/.06 (1 – 1.06^{-75}) = $47,869. He did not elaborate upon why the difference between the lease cost of 75 years was only some $600K less than an outright Fee Simple purchase of the land – but, after all, 75 years is a long time.

Note that A) = B) + C) … i.e. $47.870 = $44.961 + $2.909. This is, of course, just as we would expect. It does not matter how many time segments we split the total lease annuity into, as long as the annual cash flow and the discount rate remains the same, the present value of all the segments will always add to the total 75 year annuity.

Note, as well, that the present value of the 30 year lease, not set to begin for another 45 years, is $2.909 million (coincidentally the same amount as the annual annuity). This means that, in order to fully compensate the land owner to bind himself into this 30 year extension, and have that commitment effective right now – today, would require a one-time immediate prepayment of $2.909 million. Such an outcome is not the mere manipulation of a few discounting formulas, but a highly defensible financial certainty given the acceptance of the estimated market value purchase price and the applicability of the 6.0% discount rate.

So how did the property appraiser come up with the $14.414 million value? This prepayment would appear to be a windfall gain in the amount of $11.505 ($14.414 – $2.909) to the Lessee and a disadvantage to the Lessor. It is apparent that the appraiser is not aware of the contradiction he has perpetrated against his own logic.

IMPLICIT FUTURE APPRECIATION IN CURRENT MARKET VALUE

In attempting to understand the appraiser's conclusion, one comes to realize that the current market prices for land implicitly include a provision for future appreciation in value. Historically land values continue to climb a slow and steady path upwards[3] and that any purchaser of that property will expect to pay more for the fact that it is an appreciating asset. Conversely, any landowner who leases out (rather than sells) his property will recognize that there is an opportunity cost involved in long-term fixed-rate

[3] Here 'slow and steady' refers to the long-term geometric average return on land appreciation. Of course, during boom years land prices respond accordingly, but these spurts tend to be mitigated during years of recession.

commitments. That opportunity cost is the benefit forgone of being able to re-price the lease rate in step with the actual appreciation of the land as time goes on.

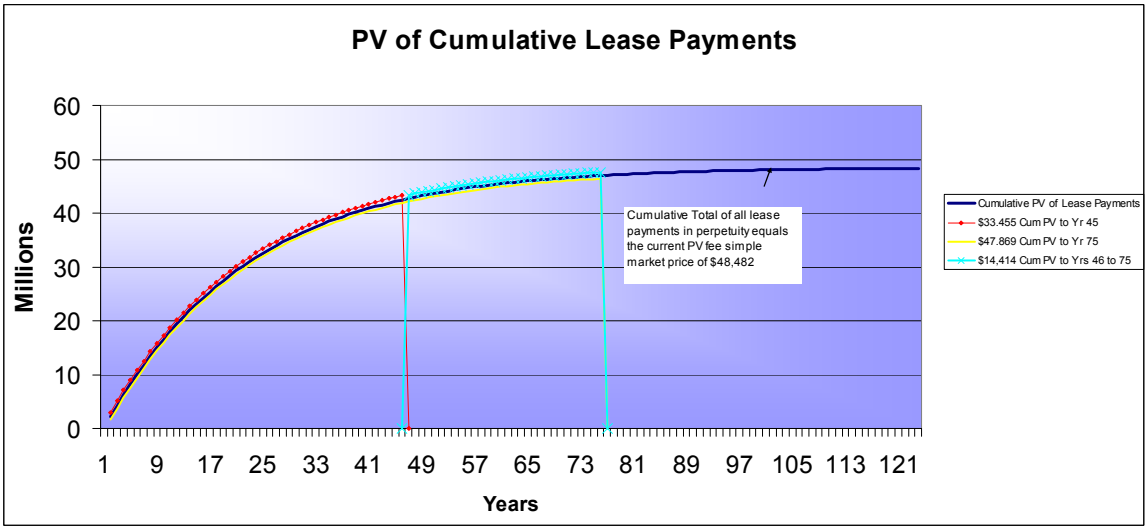
After all, if, for example, a Lessee requires 6.0% return on his 10 million dollar property he therefore would be willing to accept a long-term annual rent of $600K. Now, if the property value unexpectedly increases to $11 million he may still be getting a nominal 6.0% return on the historical value of his capital investment, but is only earning a 5.45% ($600/11,000) return on the opportunity cost of the investment.

THEORETCIAL APPROACH TO PRICING FUTURE APPRECIATION INTO LAND LEASES:

One possible method of pricing in the future appreciation expectations into a long-term fixed rate lease (and, a possible explanation of why the appraiser's opinion in the current case is so divergent from otherwise reasonable expectations), would be to assume that the current market price of land is the aggregate of a base 'no-growth' amount plus a provision for all future appreciation. And even though fixed-rate long-term annuity type leases may still be found in practice, these implicitly are priced to compensate for the increasing opportunity cost of the underlying asset[4].

In the current case under investigation, we are attempting to reconcile the cumulate PV's at four differing points in time:

[4] Credit risk and reinvestment risk notwithstanding, it is only the Present Value of any stream of cash flows that matters. So, two Leases of the same term and very different cash flows but equating to the same PV can be thought of as the same. In the case under consideration, the 30 year term is different from the base-case 75 year term, and therefore it is very important how the cash flows have been scheduled.



Namely, there may be a function that shows cumulative total PV (i.e. area) of:

1. $48.482 under the entire curve (i.e. perpetual lease payments)
2. $47.869 under the curve between the years 0 to 75 inclusive
3. $33.455 under the curve between the years 0 to 45 inclusive
4. $14.414 under the curve between the years 46 to 75 inclusive

Such a design is implicitly what the property appraiser has suggested in his opinion. Once we discover exactly what the mathematical function is that would produce such a curve, we would also have learned precisely which growth profile had been projected throughout the life of the lease payments.

Devising the necessary formula is relatively easy[5]:

$$\text{PV of Land} = C\left[\left(\sum_{n=1}^{75}\right) e^{(g - Disc)\,t}\right] + \$48.482\, e^{(g - Disc)\,75} = \$48.482$$

Where:

C = the initial annual Cash Lease Amount, (pre first years growth)

g = the implicit constant annual rate of growth / appreciation in the

[5] Here we are using continuous compounding rates for the "g" growth factor and discount rate of 6.0% [which equates to a continuously compounded rate of 0.0582689) simply because it is easier to work with these natural log exponents.

land value

Disc = the annual compounded discount rate of 6.0% or 5.82689% as stated in a continuously compounded rate

and

PV of Lease Payments,

Yrs 0 to 75 Inclusive = $C\ [(\sum^{75}_{n=1}) e^{(g - Disc)\, t}] = \$47.869$

and

PV of Lease Payments,

Yrs 0 to 45 Inclusive = $C\ [(\sum^{45}_{n=1}) e^{(g - Disc)\, t}] = \$33.455$

and finally,

PV of Lease Payments,

Yrs 46 to 75 Inclusive = $C\ [(\sum^{75}_{n=46}) e^{(g - Disc)\, t}] = \$14.414$

However, actually completing the integration by parts in can be notoriously challenging for those of us with only rudimentary calculus skills. Fortunately, there are a number of shortcuts that can be applied first in order to test if such a function does actually exist.

USE OF THE GORDON GROWTH MODEL

For example, as an initial reasonably test, one can presume an acceptable value for g, the constant annual rate of land value appreciation, and then apply the Gordon Growth Model[6] in order to arrive at the precise value for the no-growth 'base-value' of the land. A quick investigation into the long-run history of land prices in the area suggested a 1.2% geometric average annual appreciation had been experienced in the past and may be a reasonable estimation for a perpetual rate of future growth. Assuming, then, that this is the "g" in the Gordon Growth Model[i]:

[6] The Gordon Growth Model is most frequently used to assess the value of shares that are believed to be subject to a constant and perpetual rate of growth. Share Value = D1/(r – g) where D1 represents the expected dividend payout one year hence, r is the risk-adjusted (no-growth) discount rate and g is the constant rate of annual growth expected.

$48,482 = C / (0.06 – 0.012) Where "C" = the initial Lease Amount, with one year's growth built in

C = $2.327

So, if $48.482 is the combined value of the Land with an annually expected rate of 1.2% growth, and $2.327[7] would be the risk-adjusted initial lease payment expected at the end of the first year (and this, in turn, would be expected to appreciate at the rate of 1.2% each year to eternity), then we can re-apply the Gordon Growth Model in order to determine what the 'non-growth' value of the land would be:

No-Growth Base Value = C/(0.06 – 0.00) = $2.327/0.06 = $38.783

Therefore, we can now speculate that the current market price of the property consists of a no-growth base amount of $38.783 million plus a $9,699 provision for future appreciation ($48.482 - $38.783). Further, since we have solved for the initial "C" lease payment at an assumed 1.2% constant growth rate, we can plug these values into the function for the PV of the Growth Lease Payments assumed to occur between the years 46 to 75 inclusive:

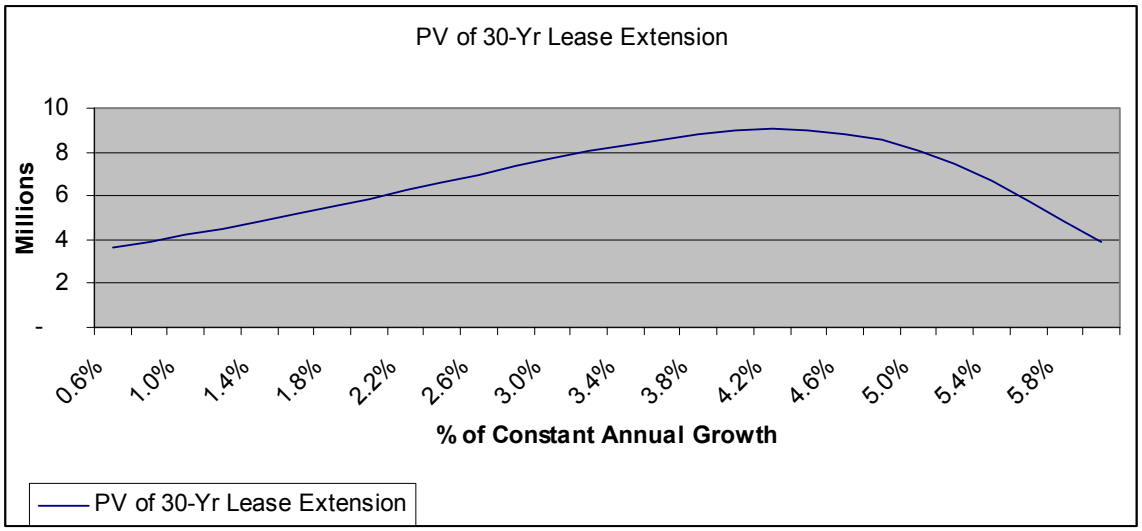
PV of Lease Payments,
Yrs 46 to 75 Inclusive = $C[(\sum^{75}_{n=46})e^{(g - Disc)\,t}]$

$PV = [\$2.327/1.012 \sum^{75}_{n=46} e^{(0.011928571 - 0.058268908)\,t}] = \$4.524$

RECAP: In an attempt to explain the property appraiser's significantly different PV of the 30 year future lease prepayment, the theory has been offered that implicitly the future value appreciation on real property (land, particularly) is included in the market's assessment of its current price. Even though a long-term lease may be framed in a stream

[7] Note that $2.327 is LESS that the $2.909 Fixed Annuity that had been presumed in the initial 75 year Lease. This is consistent with the theory put forth that, land values are expected to increase throughout the term of the lease and all other things being equal, the annual lease payments would be expected to appreciate accordingly.

of constant fixed-amount lease payments, the underlying economics are such that these compensate for the expectations in future growth[8]. In putting this hypothesis into practice, we have found we have explained a little of the difference between the $2.909 prepayment the appraiser's fixed-annuity method would have suggested. Now, we can reasonably support a one-time prepayment of $4.524 million. This is, however, still a long way from $14.414 that the appraiser concluded upon. The question must be asked, therefore, is there any possible amount of constant annual growth that would explain the $14.414 conclusion? The following is a graph shows the outcomes for the aforementioned 30-Year Extension Function, at various growth rates:



Note that, while initial increases in the annual growth rate assumption leads to higher PV's for the 30-Yr prepayment value, eventually (at 4.2%) this trend peaks and begins to decline. This is so because, even through the annual lease payment continues to grow, in compensation, the starting point, the no-growth base assumption, must correspondingly fall in order to make the entire steam of lease payments equate to the known $48.482 million[9]. Another way to conceptualize this balance between future growth expectations

[8] In this case particularly, the fixed annual annuity amount was just a convenience by which to arrive at a PV, as the intent was always that the lease would be prepaid in advance via one lump-sum.
[9] It is an axiom of value that, since the actual market purchase price of the land is known with relative certainty, the PV of all the lease payments must always sum to this amount. It would never make financial

and the non-growth base amount is to consider what happens as future expectations become more and more exorbitant. As future appreciation expectations begin to climb to 5, 6 and 7% annual so must the current market price of the asset – people bid up the current prices of assets that have increasingly stronger expectations of future growth. In our model, however, we already know that the $48.482 market value represents the current price, therefore, increases in future growth must be offset by a corresponding decrease in the no-growth base amount.

AN ALTERNATIVE SUPPOSITION TO THE $14.414 PV

Given the strength of the argument advanced so far it is just not possible to believe the $14.414 30-Yr prepayment is a supportable estimate. This statement is contingent, however, upon the strength of the $48.482 estimated market value of the property, and the appropriateness of the 6.0% discount rate. Both of these were strongly supported in the analysis provided by the appraiser, and on the strength of that we have accepted those as inviolate. So what then might support a PV of $14.414?

If we accept the $14.414 as correct, it can be shown that this must mean that the value of the land is expected to grow to a nominal amount of $240.229 million[10]. This is a factor of 4.955 from the currently estimated market price (i.e. $240,229/48.482 = 4.955 or 395.5% cumulative growth). This in turn represents an annual constant rate of growth of 3.62% ON THE ENTIRE $48.482 current base ($48.482 x (1.0362^{45} = $240.229).

So, if we are to accept the $14.414 PV of the 30 Lease Prepayment, we need to believe that, on average, the existing market price of land is expected increase by a full 3.62% each year over the next 45 years – and somehow the market has not priced this

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sense that a lessee would knowingly commit to a stream of long-term lease payments that exceeded purchase price of the asset.

[10] $14.414 current dollars restated to a future value 45 years hence is $198.403 [$14.414 \times 1.06^{45}$]. Then, if we ask ourselves, what kind of 30 year annuity would equate to a PV of $198.403: $198.403 = C/.06(1 – 1.06^{30}, we find C = $14.414 annual lease payment. Finally, since 6.0% is still the appropriate return on the property, an annual 6.0% return of $14.414 indicates the principal invested is $240.229 (i.e. $14.414/.06 = $240.229]

tremendous potential into the existing market price and bid it up accordingly[11]. Perhaps the appraiser is the only one who can identify this trend?

CONCLUSIONS

Implicit in every current market price for appreciating assets is an estimation for the future growth factor. If the rate of return that the market is currently expecting on those types of assets is known, then the Gordon Growth Model may be applied in order to estimate a "no-growth" base amount. This in turn is used to test the reasonability of various constant-rate growth profiles.

In the specific case analyzed, the $14.414 PV Prepayment could only be accepted as reasonable if it is assumed an annual increase in asset value of 3.62% will occur in each year for the next 45 years and NONE of this aggressive price appreciation has currently been priced into the existing market value.

[i] An interesting conceptual issue arises at the point where the denominator of the Gordon Growth formula is considered. We have speculated that the 6.0% annual fixed annuity of $2.909 return on Land is actually a hybrid shorthand averaging for both the non-growth base-amount of the asset, plus a provision for all future capital appreciation expectations for that asset. Does this mean that the 6.0% represents just "r" in the Gordon Growth Formula, or is 6.0% = r – g? We have speculated that, because the returns of $2.909 are restated in a flat, fixed-rate return, that 6.0% must equal just "r". This is also consistent with the appraiser's implicit findings stating that, in relation to a fixed historical capital investment, a fixed, non-appreciating annual return of 6.0% is appropriate.

[11] It is tempting at these junctures to start assuming such contradictions in are caused by non-quantifiable subjective reasons. Such as 'While the future appreciation is as significant as 3.62% annual for 45 years, this has not been priced into the current market estimate because the land will not be a liquid investment … or the use of the land is expected to be highly restricted.' The problem with this reasoning is that the market would incorporate all those subjective factors into the current price. Illiquidity and Restrictions on Use would be reflected both in the current market price as well as any potential for future growth.