



ON THE PRACTICALITY OF USING A SINGLE POINT-ESTIMATE OF THE DISCOUNT RATE IN LONG-TERM DISCOUNTED CASH FLOW VALUATION

By Richard R. Conn CMA MBA CPA

EXECUTIVE SUMMARY

- Discount rates used in business valuation are point-in-time estimates of long-term risk-adjusted rates of return. They are not capable of capturing changes in value that will/could occur as a result of unforeseeable future economic events.
- Foreseeable changes in the future risk profile should be represented in the current discount rate.
- It is possible to represent all the foreseeable future risk changes into one overall blended discount rate and still obtain reasonably reliable results

INTRODUCTION

In business valuation the determination of present value of any business or asset is often arrived at from the discounting of expected future cash flows. The ‘discount percentage rate’ is meant to reflect:

1. the pure real time value of money
2. the expected inflation impact¹, and
3. the inherent riskiness of the estimated cash flows.

A great deal of importance, therefore, is placed upon the selection of the discount rate as this single percentage can have a profound impact upon the outcome of the valuation.

¹ Items 1. and 2. combined can be considered to be the nominal risk-free rate of interest and are analogous to the opportunity cost of money. Any investor purchasing an asset or business would, at the very least, expect to earn the same return as that available to them had the money been invested in government bonds.



AN EXAMPLE

Take, for example, an asset that is expected to generate a perpetuity of \$100 at the end of each year where the discount rate has been determined to be 10.0%. The current valuation of such an asset would be calculated to be \$1,000². If, however, the assessment of the 10.0% was wrong just by 2.0% and the ‘true’ discount rate was only 8.0%, then the corrected asset value would be \$1,250, meaning that the errant valuation had been 20% too low. And, to demonstrate the extreme sensitivity to discount rates, if instead the correct rate was actually 12.0%, this would lead to a current asset value of \$833 – meaning the original estimate of \$1,000 was 20% too high.

CAN ONE SINGLE POINT-ESTIMATE OF DISCOUNT RATE BE CORRECT?

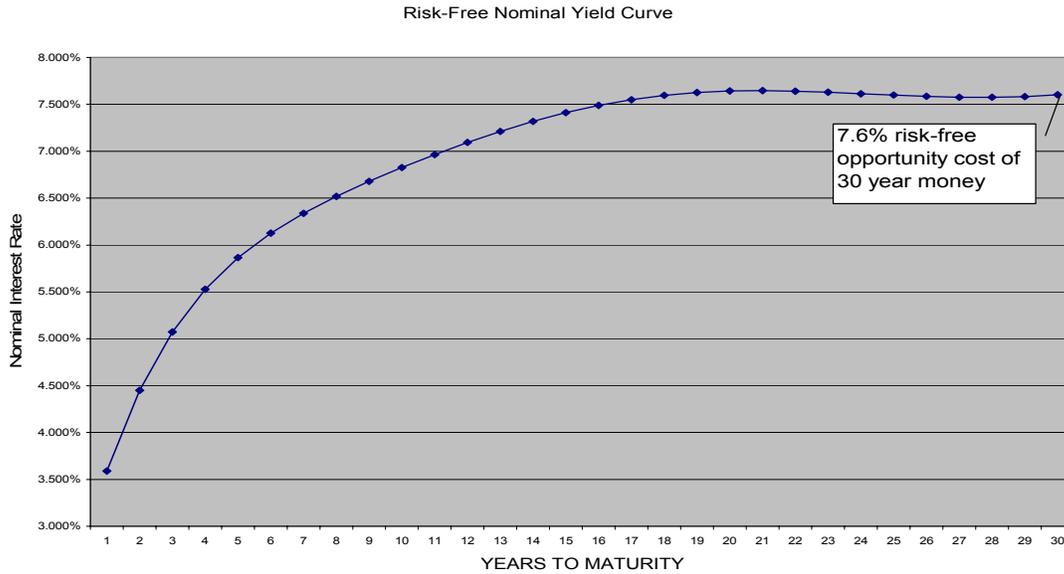
For the determination of value of long-lived assets, say one of several decades, one has to wonder if it is not tremendously presumptuous of the valuator to be firstly suggesting that one single point estimate of the correct discount rate should apply and secondly that (s)he would have the ability and foresight to assert what that rate should be.

Consider, for example, an asset that is believed to have a remaining economic life of 30 years being valued at a time when the nominal (meaning, inclusive of both the real interest rate as well as a provision for expected inflation) risk-free rate on 30 year government bonds was 7.6%. And, in addition, it was determined that the risk premium required for that specific asset was 5.0%. The applicable discount rate would be 12.6%.

² Using the standard formula for calculating a perpetuity of: $C/r = \text{Present Value}$, where “C” is the constant perpetual cash flow and “r” is the discount rate. $\$100/0.10 = \$1,000$

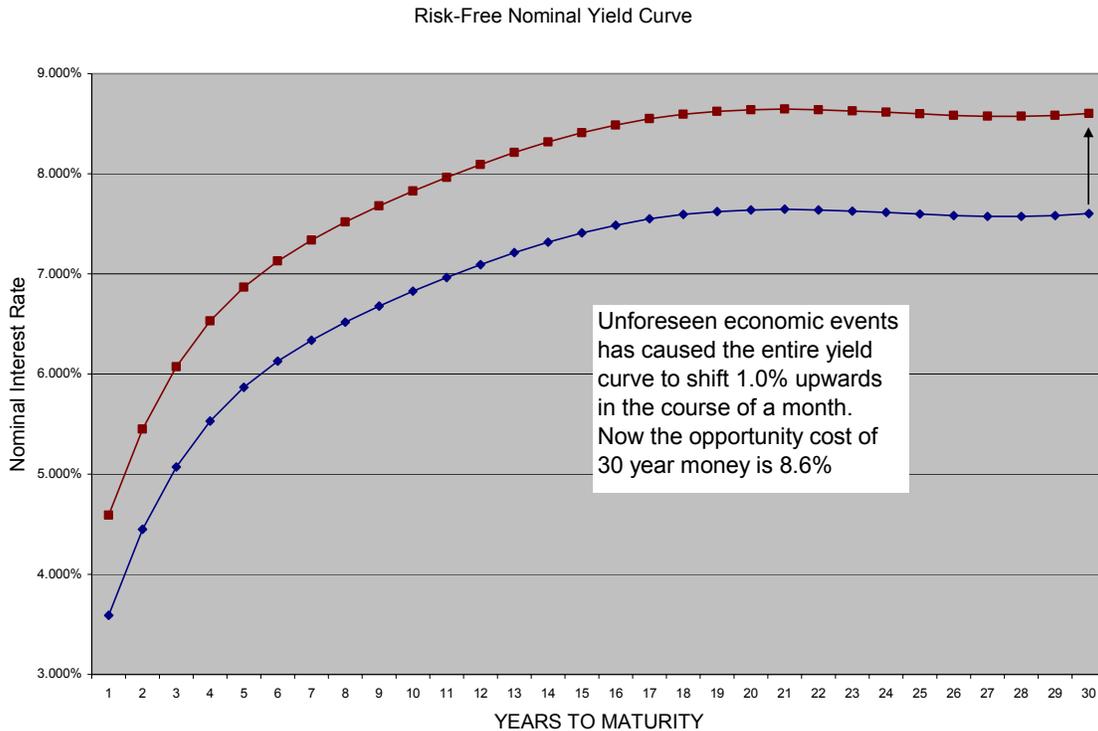


RISK-FREE YIELD CURVE³



Consider that a month goes by and, as a result of significant changes in the overall economy the entire risk-free yield curve shifts upwards by 1.0%. Now the applicable discount rate is 13.6% and the valuation of the asset has been substantially decreased in spite of the fact that neither the expected future cash flows nor the overall risk profile of the asset has changed.

³ The curve depicted here represents actual zero coupon Bank of Canada yield data as at January 15, 1997. The data is unique in that it shows the largest 1-to-30 year yield differential (more than 400 basis points) in all the historical years that The Bank publishes the zero coupon data. The yield curve also turns negative after year 21 (i.e. it would be more costly to borrow 21 year money than for 30 years). In spite of this aberration, a 30 year discount rate would still employ the lower 30 year risk-free rate as this was truly representative of the opportunity cost of that term.



POINT IN TIME ESTIMATE

This example highlights an important aspect of discount rates. They are based upon point-in-time assessments of known and expected economic conditions. As has been discussed, discount rates include the current risk-free rates (i.e. the opportunity cost of no-risk investing) which, in turn, assimilate all that is presently known or reasonably predictable about future events. As time passes, however, actual economic conditions may prove to be different from those predicted and it is these unforeseen changes that could cause discount rates to vary, sometimes substantially, from one period to the next.

We need to be careful, therefore, in the assessment of any given discount rate only to be considering point-in-time information. It would be a mistake, for example, to conclude that the 11.6% rate first determined above was in error simply because a month later the new circumstances would require a 12.6% discount rate. At the determination of the 11.6% rate (presumably at the Valuation Date), it could not be foretold that the upward



shift in the risk-free yield curve was imminent. If this were the case, the market surely would have already priced in the predicted rate increase into the current rates. Discount rates, then, must be determined without the benefit of hindsight.

Similar to the nominal risk-free rate, the risk premium portion of the discount rate will also vary. That is, as time progresses it is entirely likely that the risk premium will increase and/or decrease in response to the idiomatic risks specific to that asset as well as in response to industry changes⁴. But these are not point-in-time changes. If they are not foreseeable on the specific date at which the discount rate must be derived (e.g. the Valuation Date), they would not be represented in the overall risk premium at that point in time (i.e. one must wait for time to pass and new circumstances to become evident).

There are, however, point-in-time estimates where it would be foreseeable that the risk premium would be expected to change. For example, if it is known on the valuation date that a competitor's (Firm B) plant will be completed and begin production three years hence, this may be cause to believe that Firm A's risk premium should be lower during the first three years than subsequently. Conversely, if it is known that a competitor's (Firm B) very favourable supply contract is due to terminate after the next three years, this may be indicative of a circumstance where Firm A's discount rate may be required to be lower after three years than it is currently assessed to be⁵. If these foreseeable changes in the risk environment are substantial, the valuator may find it necessary to use several different discount rates tailored to each specific time period.

On the other hand, if the foreseeable changes in future risks are relatively small then it may be more desirable to reflect all these expected risk profile changes into one overall discount rate that is meant to accommodate all the known and expected risk variations.

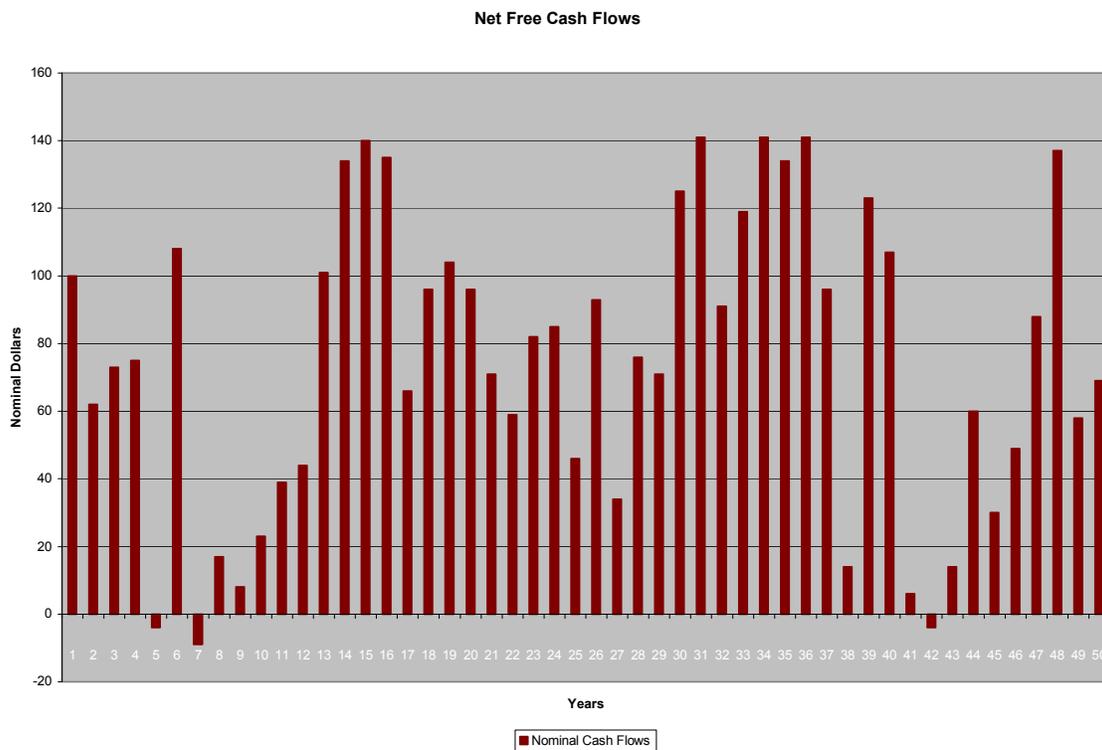
⁴ For example, technological obsolescence can cause wholesale changes to the risk exposure of any given firm. Consider how the discount rates of the VHS video tape manufacturers must have changed upon the introduction of the DVD.

⁵ The approach of incorporating these types of risks into the discount rate assumes that it would not be possible to reflect the net impact firstly into the estimated cash flows. Generally it is better if a change in the competitive environment (as in these examples) can be quantified in the estimated cash flows. Only if this cash quantification is not possible would it be necessary to incorporate these changes in risk into the discount rate.



This single blended discount rate would take into account the impact of the expected changes in the future risk profile.

So, the variable nature of risk premium in general begs the question ‘Can any single point estimate of a discount rate yield reliable results?’ The answer depends somewhat upon the expected variability in the underlying cash flows as well as the range of deviations expected in the discount rate. However, in general, it can be shown that a single point estimate of discount rate is surprisingly robust. In order to illustrate this assertion, we will assume a given asset has a 50 year life span during which it is expected to generate the following cash follows⁶:



⁶ The post year one nominal cash flows have, in order to avoid bias, been estimated via a random number generator set to accept any integer between -\$11 and \$151. These are meant to simulate the net free nominal cash flows of any business with a reasonable degree of annual earnings volatility and limited downside risk (consider the units millions of dollars if that makes the results seem more significant).



Discounting all of these 50 cash flows at an effective discount rate of 10% per annum gives a Net Present Value (NPV) of \$651. If, however, the valuator had erred in the selection of the 10% rate, and the ‘correct’ rate was only 8%, then an NPV of \$824 would have resulted. This would be 27% higher than the 10% NPV results. Conversely, if the ‘correct’ discount rate should have been 12%, then an NPV of only \$535 would have been calculated and this is 18% less than the 10% NPV results. So, using constant discount rates of either 12% or 8% produces an NPV range from \$535 to \$824 or \$289 – quite significant. Advising a client that they should be paying anywhere from \$535 million to \$824 million to acquire a business would, at the extremes, be telling them that they have the opportunity to either be paying a 54% premium over true value or obtaining a 35% bargain. In either case the advice would not be very helpful.

In contrast, what if the valuator is convinced that the 10% discount rate is the generally appropriate rate over the life of the asset (because extensive study of this firm’s most comparable competitors have been found to be valued with a 10% return by the market⁷), but expects that this will fluctuate from 8 to 12% inclusive in any given year?

One reasonable approach to simulate such an outcome would be to allow the discount rates in any given year to vary randomly from 8 to 12%⁸. Such a methodology was employed and the first NPV obtained was \$641. This compares to the constant 10% discount rate NPV of \$651 ... or only a 1.5% reduction from what would be considered the ‘true’ NPV.

⁷ Appendix A speaks to the methodology and reliability of assessing rates of return from market data.

⁸ This methodology probably produces far more ‘leaps’ in rates that normally would be observed in practice. This is because it is just as equally probable, in the experiment, for the rates to jump from 8% to 12% in any sequential year (or vice versa). In real life, however, it is far more likely that these rates would follow a Brownian motion whereby the outcome of any given year is random to, but dependent upon the outcome in the previous year. This is not to infer that occasionally in actual market conditions there will not be economic events or idiomatic circumstances that cause large shifts in discount rates between two sequential years.



In fact, the same trial conducted 100 times produces a range of NPV's whereby the lowest obtained is \$615 and the highest is \$702⁹. This falls within -5% to +8% of our predetermined 'true' NPV. A margin for error of -5%/+8% would not be considered significant in most valuations.

CONCLUSIONS

By quite simple¹⁰ (certainly non-scientific) empirical means we have shown that point estimates of discount rates can be reliable when they are intended to represent a long-term blended average rate of the fluctuating short-term point-in-time expected rates.

⁹ The standard deviation of the 100 observations is \$16. While none of the math has been included here, it can be shown, via an iterative process, that an NPV of \$615 can be derived using a constant discount rate of 10.54% and an NPV of \$702 is derived from a constant discount rate of 9.32%

¹⁰ In this simplistic approach we have not considered the potential correlation between cash flow estimates and short-term discount rates. For example, during times of economic uncertainty, it is likely that a decrease in estimated cash inflows will correspond with an increase in short-term risk-adjusted rates of return. Conversely, it is likely that economic booms experience increased cash estimates and lower risk premiums.



APPENDIX A

A More In-depth Consideration of Reference Discount Rates

In this paper the inference was made that a 10% blended discount rate was the most appropriate for the business under consideration because a number of its competitors were observed to be incurring a 10% cost of equity (presumably this observation was made using a CAPM-based approach). Such a determination by comparables is often made in actual practice. Generally, however, there are no two businesses that are exactly alike and using one firm's cost of equity as a proxy for another firm is fraught with difficulties. At best, using comparable firms can provide reference discount rates that will show a range of possible rates that may be applicable to the subject firm.

The problem of determining a current discount rate is even bigger than just not being able to locate reasonably comparable firms, however. In truth the discount rate (which in this context, for simplicity, we are presuming is synonymous with the cost of equity rate) is rarely, if ever, plainly visible just from publicly available market data. This is a surprising admission considering the markets price tens of thousands of publicly traded common equities every trading day.

Of course, one can take a CAPM¹¹ approach. This uses the historical covariance of the security's past price movements in relation to movements of the market as a whole and derives a Beta. Then, the Beta is multiplied by an Equity Risk Premium (ERP) which itself is often a very long-term indication of how much more the equity markets paid in excess of the risk-free returns. Often the ERP is an eighty year average. So, both the derived Beta and the ERP are long term averages. While there may be a great many advantages to employing a CAPM derived discount rate, there is no reason why it would reflect very current market sentiment. When, for example, a firm has undergone recent changes that are unlike any it experienced during the Beta sampling period, the CAPM results can be inapplicable to current circumstances.

¹¹ Readers are assumed to be familiar with the basics of the Capital Asset Pricing Model (CAPM)



Consider that the valuation of any firm or asset can always be represented by a very simple three variable equation:

$$V = C/r \text{ (i.e. Value = Cash / discount rate)}$$

And more meaningfully this expands to

$$\text{Value} = \text{Net Cash}_t / (1 + r)^t$$

Generally, since there are more than one net cash flows, there will be a series of Net Cash flows at times t , and, equally, a series of $(1 + r)$ discount factors at times t . In the most simplistic case, however, where there is only one Net Cash flow and assuming that the opportunity cost and risk premium of that cash flow is zero, then:

$$\text{Value} = \text{Net Cash} / (1 + 0)^t$$

$$\text{Value} = \text{Net Cash} / 1$$

$$\text{Value} = \text{Net Cash}$$

Literally, this tells us that the value of any given venture or asset would be worth the future net cash flow expected to be derived from it. This should be intuitively obvious and is a mathematical tautology, and yet there is something people find mysterious about adding up discounted cash flows that erroneously leads them to believe that Value is somehow not a direct function of the future expected cash flows. Practically, the exponential series operation does complicate the issue and if we did live in a simplistic (but wholly impractical) world where “ r ” always equaled zero, the valuation process would be considerably simpler. For example, investors would never be willing to pay \$100 for an asset/business where the sum of the future nominal Net Cash flow(s) only amounted to \$90.

Recognize that the “ r ” is the discount rate and the variable of interest of this paper. Note, however, that the “ r ” is not discernable from market data. Usually, when considering the



stock markets for example, only the current market price of any equity (i.e. the “Value” part of our equation) is publicly visible. But knowing only one term of a three variable equation does not allow one to determine the other two unknown variables.

If firms publicly disclosed their expected net future cash flows *and* if all investors accepted those estimations as reasonably the most probable¹², then it would be quite a simple matter to determine what the market accepted discount rate was. For example, if a firm was currently trading at \$50 per share and it expected a one-time only net cash inflow of \$80.53 per share five years hence, then it is a relatively easy calculation to determine that the market must be assessing that firm with a 10% discount rate. That is:

$$\$50.00 = \$80.53 / (1 + 0.10)^5$$

Occasionally firms do disclose projections of their future estimated net cash flows – but this is no guarantee that investors have accepted these estimates verbatim. So even when there apparently are two of the three valuation variables available in which to uncover the implied discount rate, the answer derived may not have any similarity with the actual “r” investors had in their heads when they decided to place a specific value upon a given share.

TAKE A 'RATE OF RETURN' SURVEY

One might believe that an overall consensus of market opinion may be had simply by polling all the existing and potential buyers and sellers of a given asset (or security) in order to determine where the balance of discount rate expectations lay. However, the rate information alone would not be conclusive without also enquiring on the net cash flow

¹² The requirement of this second condition should not be underestimated. Even when firms go to great lengths in providing long-term cash flow expectations in their prospectuses, The Market (e.g. independent financial analysts and large institutional investors) will not accept them at face value and create their own cash estimates. The presumption is that, no matter how strictly regulated, no firm self-reporting upon its own future will be entirely free from bias.



expectations of each buyer and seller. And, there may be as many unique cash expectations as there are potential investors.

For example, take the instance where two investors agree that a given asset will only provide five future annual net cash flows before expiring. Further, both Investor A and Investor B claim that they agree the appropriate risk-adjusted return for that asset is 10% and that the current Value of the asset is \$379. When asked to disclose their cash flow expectations, however, it is discovered that Investor A presumes a constant annual return of \$100 whereas Investor B believes that cash flow will initially be zero and gradually increases in each successive year thereafter.

For example:

INVESTOR A

$$\$379 = \$100/(1 + 0.10)^1 + \$100/(1 + 0.10)^2 + \$100/(1 + 0.10)^3 + \$100/(1 + 0.10)^4 + \$100/(1 + 0.10)^5$$

INVESTOR B

$$\$379 = \$0/(1 + 0.10)^1 + \$45/(1 + 0.10)^2 + \$105/(1 + 0.10)^3 + \$175/(1 + 0.10)^4 + \$231/(1 + 0.10)^5$$

A cursory consideration of the differences in cash flow expectations might mistakenly lead one to believe that they do not matter. Assume, for example, Investor A was informed that his expectation of a constant cash inflow is incorrect, and that Investor B's projection was much more probable. So what – we say? Obviously, if Investor B's cash expectations are substituted into Investor A's valuation formula, the same \$379 will result. As long as both investors believe a 10% risk-adjusted return is appropriate, either cash outcome will yield the same \$379 value expectation.

But the reality is that Investor A, who was expecting a very predictable unchanging return of \$100 per year is unlikely to perceive the other cash projection as reasonable



substitute. Investor B's cash expectations are riskier in that they occur later in the asset life and have a much broader range (\$0 to \$231). When faced with this potential cash payback, it is highly likely that Investor A¹³ will require a greater risk-adjusted return in order to induce him to make the original investment.

Once properly informed as to the cash flows, Investor A will probably reject 10% as too low a return given the risks involved and may, instead, for example, demand a 14% return. And, at 14%

$$\text{\$329} = \$0/(1 + 0.14)^1 + \$45/(1 + 0.14)^2 + \$105/(1 + 0.14)^3 + \$175/(1 + 0.14)^4 + \$231/(1 + 0.14)^5$$

this asset declines in value to \$329. In other words, based upon an originally inaccurate perception about the potential cash expectations of this asset, Investor A would have paid \$50 in excess of his own risk tolerant price level.

This example has demonstrated several important discount rate factors:

- Each investor has his own personal tolerance for risk and that will determine the purchase or sales price of every transaction they engage in.
- When valuers speak of “the” discount rate it is the one that would represent the requirements of the notional buyer and seller. That is, the overall market consensus all the buyers and sellers trading in that asset¹⁴.
- Even when it appears that bona fide purchasers concur upon the risk-adjusted discount rate associated with a given asset, there can be no real assurance of this fact until the cash flow expectations of each purchaser is identified and compared.
- Discount rates are elusive and cannot be derived simply by observing publicly available market data.

¹³ The fact that both investors were willing to accept a 10% return given their substantially differing assessment of cash flows indicates that Investor B has a much higher risk tolerance than does Investor A.

¹⁴ Attempting to determine the market consensus for assets where there are only a few buyers and sellers (e.g. high-end fine art or expensive yachts) is highly problematic. As we have seen, the risk tolerances of the individuals involved can vary greatly making determination of an overall market average discount rate much more tenuous than an asset which is traded by thousands of people every day (e.g. common shares).



And, on a broader scope, this Appendix has also served to highlight the following important observation:

- Traditional methods of deriving a discount rate for any asset are usually based upon historical pricing data. To the extent that market circumstances and other historic issues differ from the present day environment, so too will be the potential for these methods to fail to properly reflect current investor sentiment.