Accession Capital Corp

APPLICATION OF BLACK'S MODEL TO DETERMINE RATE OF RETURN FOR A HYBRID SECURITY

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EXECUTIVE SUMMARY

- Assignment was to recommend the appropriate risk-adjusted return applicable to a hybrid security issued by a mid-sized income trust
- Proposed risk profile was unique to security in that it would face all the down-side risk of a common equity, but have a limited upside potential similar to a variable-rate preferred share
- No similar public security was available upon which to draw inferences of return demanded by the market
- An adaptation of Black's Model was applied in order to simulate the security's risk profile
- It was concluded that the security should pay a maximum floating return of CDOR + 9.5% (stated annually)
- Relatively simple derivative models can be used to identify and rationally support rate of return hypothesis that might otherwise only be arrived at by purely subjective reasoning

OVERVIEW OF THE ASSIGNMENT

The client is a mid-sized Canadian Income Trust that had just acquired approximately $1 billion in oil and gas transportation and processing assets. For legal and tax reasons these assets were to be held by an internal Limited Partnership (LP) which would be issuing Class A and B Units. The Class A Unit Holders would be providing virtually all of the required capital with the B Unit Holder only proving a nominal amount. The A Units were to receive annual cash distributions on the basis of a blended rate consisting of CDOR plus a fixed percentage (the combined CDOR + fixed rate was referenced as the Preferred Interest Rate or PIR). Some of the salient features of the Class A Units that had significant impact upon future value were:

- non-cumulative[1]
- non-participating (any distributable cash in excess of the PIR accrued to the benefit of Class B Unit Holders only)

[1] Within a given year, the Class B Unit Holders could not receive a cash distribution unless and until the Class A's had been paid the PIR – but if the General Partner elected not to pay any cash distributions within that year, any cash that may otherwise have constituted the PIR in that year accrued to the benefit of the Class B Unit Holders in any subsequent years.

- non-voting (the General Partner retained all executive and administrative authority)
- seniority in liquidation, but never to a greater amount than the $1,000 face value/initial principal of the unit
- redeemable at $1,000
- retractable at $1,000

Our involvement in the case was twofold:

- Recommend and analytically support a Fixed Rate (to be added to CDOR) percentage that adequately compensated the Unit A Holders for the inherent risk in those securities
- State the likelihood that the future value of the Class A Units might appreciate beyond a certain percentage[2]

BACKGROUND STUDY IN MARKET COMPARABLES

The public markets are sprinkled with a few Variable Rate Preferreds (VRP's) that are, in many ways, very similar to the Class A Unit described above. In the U.S., Freddie Mac, Fanny Mae, Goldman Sachs, HSBC USA Inc. and Lehman Brothers Holdings have all issued preferred shares that pay dividends upon a floating basis (often LIBOR plus a certain percentage). In Canada, Power Corp. and Alcan have both quite successfully issued VRP's. Many of these, like the LP-Class A units under study, are non-cumulative, non-participating and ultimately redeemable at the face value amount (thereby negating any possibility for any real long-term capital appreciation).

Not surprisingly, a correlation was found between the price volatility of a VRP and the interest-reset period. 'Interest Reset' refers to how often the payment amount is recalibrated to a quoted LIBOR amount. If, for example, a LIBOR quote was captured

[2] This part of the assignment was crucial to the client, and eventually led to our recommendations of an absolute ceiling to the PIR in order to cap the maximum potential value. However, this aspect of the assignment had less to do with the application of Black's Model in arriving at the Fixed Rate portion of the PIR – which is the focus of this paper.

on a predetermined date and then held as the effective basis for dividend payment over the next two years (as was the case with some Freddie Mac securities) – those VRP's demonstrated the greatest amount of market price volatility[3]. In contrast, VRP's that reset the interest rate to the quote effective rate the day before the start of any quarter and then updated that rate each quarter demonstrated the least amount of security price volatility. In those latter cases the securities tended to stay around the face or redemption amount, gradually increasing in value until the accrued dividend payment was made and then descending back to the redemption amount. In this manner, the VRP's with very short interest-reset periods behaved very much like debt instruments. In contrast the VRP's with longer reset periods still allowed investors to speculate upon the ultimate direction of the base interest rate and thereby widening the range of the underlying security price than would have been possible with a shorter reset period. Even so, the VRP's with longer reset periods ultimately still fluctuated around the face or redemption amount of the security.

Our subject security, however, was different to all of the other publicly quotes VRP's in that the VRP's generally only constituted a small minority of the total capital contributed to the firm. In the Goldman Sachs case, for example, the VRP's amounted to some $1 billion in market cap, but this contrasted to an $85 billion in common equities for that firm. The Goldman Sachs VRP's were non-cumulative, however, there was little probability that the dividends would not be declared knowing that denying the VRP Holders their otherwise prescribed amount would also mean that no dividends could be paid to the common shareholders either.

In our Canadian Income Trust case, the Unit A Holders were to provide virtually 100% of the capital but would be subject to significant risk if the General Partner deemed that no annual cash distribution was to be made in that year. An even greater risk is represented by the fact that, in general, the Unit A Holders were to be subject to all the downside

[3] This is assuming that the creditworthiness of the VRP issuer remained at the same relative high standard throughout the period of observation – as was the case will the small sample of VRP's we examined. As a result, the primary reason for price volatility becomes speculation on where LIBOR rates were headed and how that interacted with the currently set LIBOR rate.

risks commensurate with a common equity position, but ultimately would be limited to an absolute upside payoff no greater than the PIR. Any year, for example, where there was insufficient cash generated to distribute would leave the Unit A Holders without any return whatever – whereas a subsequent boon year would still only entitle them to the maximum payout of the PIR, any additional distributable cash generated in those years would naturally accrue to the benefit of the Unit B Holders.

With these contractual restrictions in place, the assignment focused upon how to correctly model the risks that the Unit A Holders were subject to – and come up with a PIR that would adequately compensate those security holders.

RISK ANALOGY – CAPPED UPSIDE

If the Class A Unit Holders had invested their $1 billion in common stock their risk profile would have paralleled the net free cash flows of the underlying assets themselves. That is, the oil and gas assets had a determinable economic life, a defined market and a relatively predictable net cash flow what would be generated over the term of the asset lives. As such the risk accruing to the common shareholder would be dependent upon the volatility of the expected net free cash flows of the assets.

In this unique circumstance, however, the risk profile of the assets only matched the Class A Units on the downside. That is, only if the actual net free cash flows turned out to be less than *expected* would the Class A Unit Holder bear the same risk as a common equity holder[4] – this would not be true of the upside. In the event that net free cash flow turned out to be in excess of *expected*, then the Class A's, by virtue of the partnership agreement would only receive the capped PIR whereas ALL of the upside benefit would normally accrue to the common shareholder.

In essence, the Class B Unit Holder held a long-call option on the performance of the assets. If the assets generated cash in excess of a yet undefined amount, then all those

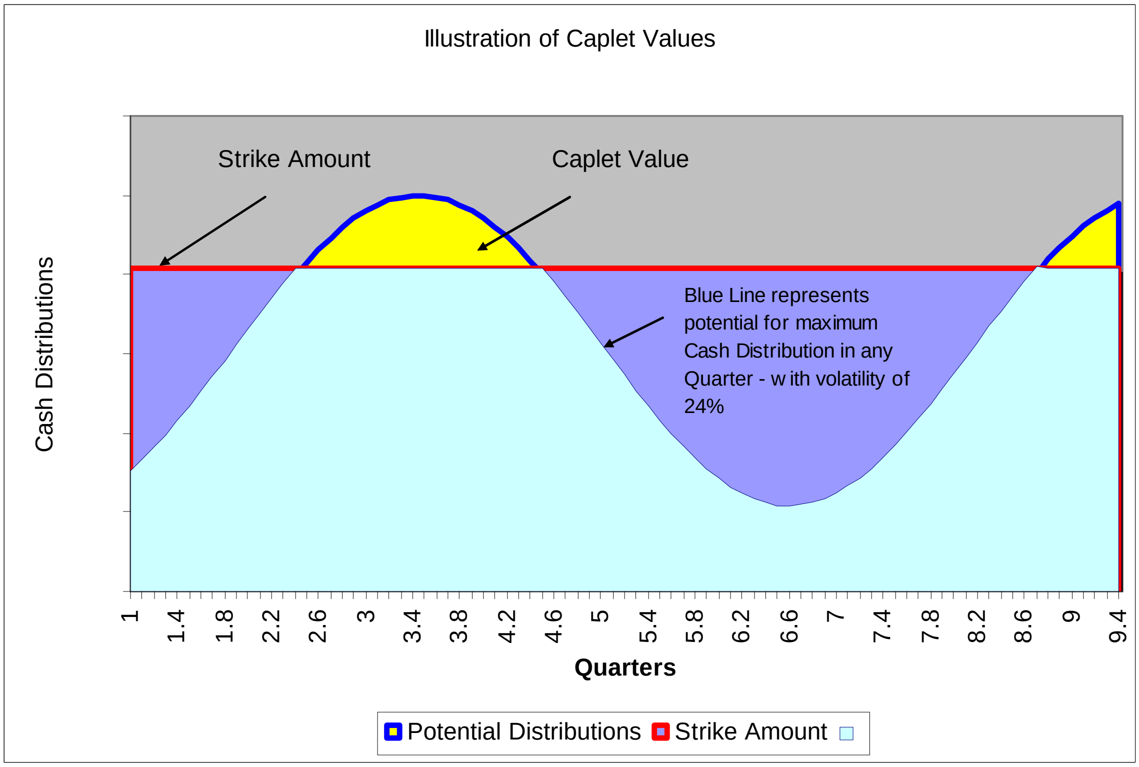
[4] The one possible except is that, in the event of a liquidation, the Class A Holder would have a priority claim to net assets, whereas a common share holder would only have a residual interest.

benefits would accrue to the Class B's. By calculating what the option price was for such a position, one might then be able to factor out what rate of return the Class A's should be entitled to on the foregone upside. There is, however, several difficulties in modeling these circumstances from a Black-Scholes type of option formula. The first is that most call option models are not continuous – they are set to be exercised only once. In our circumstances, it was theoretically possible for the Class B's to exercise an option in each of the remaining 30 years of the asset economic life. The second was that the 'strike price' (the PIR) was not pre-defined and independent of the model itself – indeed the goal was to determine what strike price should be set to. It was circular reasoning to attempt to discover an appropriate risk-rate (the PIR) with a process by which one of the primary determinants of risk was the strike-price.

BLACK'S MODEL

Unlike Black Scholes, however, Black's model was designed to be continuous (i.e. can be used to value a series of exercise instances) and could be adapted to the circumstances at hand. Black's model is most often used in the application of capped variable rate loans. Where a borrower wants to pay a floating rate (generally stated in LIBOR plus some percentage amount) but also have the security of a guarantee that limits that absolute rate that will become payable during the term of the loan. Based upon the historic volatility of the reference interest rate in question (e.g. LIBOR), the model determines the probability of the rate ascending above the preset 'Strike' rate and then calculates the present value of the interest payment that is forgone (from the lender's point of view) on all the occasions where such a rate would be exceeded.

Visually, the model can be conceptualized as the following:



While the effective rate on the loan generally floats with the interest rate index (the blue line above), the Cap (the yellow areas above the red horizontal line above) limits the total interest rate liability to a maximum predetermined amount. To the Lender, all the area above the red line represents 'Foregone Upside' that reduces profitability on the loan (compared with a similar floating rate loan where no maximum cap is set). The Lender, therefore, will wish to be compensated for the lost 'Caplet[5]' value of interest income that resides above the red line and below the blue. To the Borrower, the cap represents an insurance policy that, no matter how high the interest rate index ultimately climbs, or how often it does so – assuming the interest payments are made quarterly as we have here, he/she is guaranteed to a maximum rate payment at the red horizontal line. Like all

[5] Common terminology for the model is that all value above the red horizontal line and below the blue (the hatched area) is referred to as a "Caplet". This area is analogous to a Call on the distribution amount. Whenever the distributable cash rises above the "strike price" (represented by the red line) the call would be in the money and the long call holder would exercise their option. Here the long call holder is the Class B Unit Holders, and the writer is the Class A Unit Holders – whenever distribution amounts rise above the strike rate the Class B's exercises their call against the Class A's and recover the sum total of distributable cash in excess of the strike amount. There is a corollary to the Caplet – called the "Floorlet" and that is more salient to the case at hand. Floorlet's will be described in detail later in this paper.

insurance policies, the Borrower will expect to pay a premium for this coverage and it is this premium that Black's Model has been designed to calculate.

ADAPTATION OF THE MODEL

In the case under consideration, however, we were not attempting to value a capped interest liability on a fixed amount of principal, but rather determine a fair return on a hybrid security that was subject to a number of highly restrictive covenants.

We knew from the recent purchase of the assets that the IRR expected over the entire remaining economic life of those assets was 10.2% The vendor and purchaser were highly sophisticated, well-informed and very successful participants in their industry. The 10.2%, therefore, was a very reliable number that would have been an irrefutable proxy for the equity cost of capital on these assets. We reasoned, however, that if a 100% common equity holder would have an *expected* return of 10.2% on the investment, the unique position of the Class A Unit Holder would require and even *higher* rate of return. After all, the Class A Unit Holder was forgoing all the upside on all potential future returns. The best they could do would be to earn the PIR and, if that amount was represented as the red horizontal line in the previous graph, then all the Caplet value above that would accrue to the Class B's.

An apparent contradiction had arisen. If a credible and well-documented analysis from an industry expert had shown that the *expected* return on the underlying assets was no more than 10.2% for the remainder of their lives, how could we reason that the Class A Unit Holder's deserved *more*? Where was the additional cash to come from? After all, if the Class A's were providing 100% of the capital, didn't they just deserve the 10.2% that any common equity holder would have expected? The key to understanding this issue relied upon the fact that the PIR was not being calculated to arrive at a traditional cost of capital return that the Class A Holder's would expect in order to warrant their investment in the venture. It was, instead, an upper ceiling on the return that he should get in order to justify the unique risks inherent in that particular security. Had, for example, the assets been financed with 100% common equity and it eventually turned out that the 10.2%

return was too conservative – that a 15.2% return was *actually* generated over the entire life of the assets – then the equity holders would naturally benefit from the 5.0% of the unexpected upside. Such would not be the case with the Class A arrangement, however. So, the PIR had to be set high enough to provide the Class A's with a 10.2% return on their investment plus enough unexpected upside to compensate them for the fact that ultimately, they would no longer share in additional cash flows no matter how advantageous the actual returns became.

If, instead of a floating interest rate index, the blue line of the prior graph represented the maximum potential net free cash flows generated by the actual assets in any given quarter, then we can hypothesize that there will be a predictable normalized distribution of cash that can be modeled for the entire life of the assets. We reasoned that the volatility of these cash flows could be represented by an industry average of stock-price volatility from firms that primarily held and operated the same types of assets that had currently been purchased by our client. This industry average volatility equated to approximately 24%.

The client had prepared a detailed estimate of net free cash flows accruing to the assets for each quarter over the remaining life of the assets. These expected cash flows represented the forward price that was set for each future caplet payout. Of course, the management estimates of the expected future quarterly cash flows were stated in nominal terms. Black's model, however, like the Black Scholes option pricing model, assumes (in fact, absolutely requires) risk-neutral inputs so the nominal expected cash flows first needed to be restated on a risk-neutral basis. The risk-neutral growth rate can be derived quite easily from the formula:

$$g_N = g - ß \times MRP$$

Where:

g_N is the Risk-Neutral Growth Rate
g is the Nominal or True Growth Rate
$ß$ is the Beta or systematic risk of the risk driver and can be represented by the same Beta regressed from industry average stock prices that were used to determine 24% volatility proxy for

these assets
MRP is the Market Risk Premium for an asset of ß = 1

RISK-NEUTRAL FORWARD DISTRIBUTION CURVE

In the case in question the ß for the representative group of equities was found to be 0.89 and the MRP was 5.1% (stated at an annual rate) as reported by Ibbotson and Associates. Therefore, it was possible to arrive at a risk-neutral Forward Distribution estimate simply by discounting the expected nominal quarterly cash distribution by $(ß \times MRP_q)^t$.

Forward Risk Neutral Cash Distribution = $E[D_t] / (ß \times MRP_q)^t$

Where:

$E[D_t]$ is the expected nominal cash distribution at time t
MRP_q is the MRP stated at the effective quarterly rate

PROXY FOR CDOR FORWARD CURVE

It would also be necessary to approximate a forward curve for CDOR extending forward for each quarter of the entire life of the assets. The Bank of Canada regularly publishes a quarterly Zero Coupon Yield Curve projecting forward for the next 30 years. So, in order to approximate a CDOR forward curve for the same period, it was reasoned that the CDOR rate would normally be closely correlated to the zero coupon yield, but include a marginal risk factor to represent it's commercial status. As a result, 20 basis points was added to the zero coupon yield curve and this then became the proxy for the CDOR forwards.

The CDOR forward curve was important, of course, because it would be CDOR + X% that would define the PIR – the amount that theoretically would represent the maximum payout necessary to compensate the Class A Unit Holders for taking risk inherent in holding that security.

STRIKE PRICE IS THE DEPENDENT VARIABLE

From a Option Pricing kind of perspective, note that the theoretical PIR Cash Distribution represents the Strike Price on a traditional option. That is, in any given future quarter, a distribution of [Original Principal x (CDOR + X%)] equals the clearing point below which needs to flow to the Class A's in order to adequately compensate them, and above which is becomes forfeited upside that accrues solely to the benefit of the Class B's. Viewed in this context, the Strike Price of the quarterly distributions becomes the dependent variable in our model and X% the independent. Once we can find an X% such that the present value of all the quarterly floorlets equates to the initial principal (approx. $1 billion) advanced, then we have found the maximum rate upon which the Class A's should be compensated.

So the expected Strike Price in each future quarter then became:

$$\text{Strike Price} = \$1B \times (CDOR_{qt} + X_q\ \%)$$

Where:

$CDOR_{qt}$ is the forward CDOR for that quarter, stated at a quarterly rate
X_q % is the X%, stated at a quarterly rate and determined by an iterative trial and and error process

BLACK'S MODEL – FLOORLET VALUE

The typical presentation of Black's Model is:

$$\text{Floorlet Value} = tLe^{-r(T+1)t}\ [R_x N(-d_2) - F_k N(-d_1)]$$

Where:

t is the compounding reciprocal per year (0.25)
L is the initial principal ($1B)
e is the mathematical constant 2.71828, base of the natural log
r is the risk-free rate (expressed in continuous compounding)
T is the number of periods to the maturity date of floorlet
R_x is the Strike Price/Rate
F_k is the Forward Price/Rate

In our adaptation, then, R_x is $1B x (CDOR$_{qt}$ + X_q %) and F_k is the Risk-Neutral Forward Distribution amount as previously described.

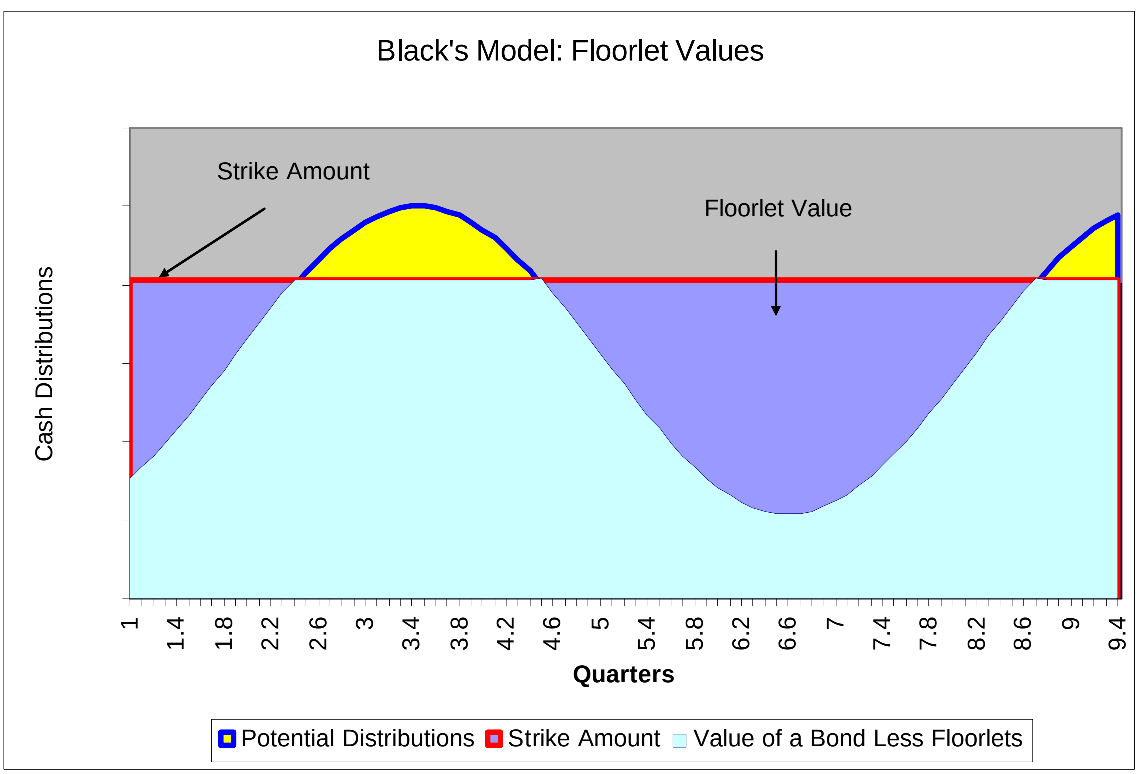
Moreover,

$N(-d_2)$ is the Normal Probability function of $-d_2$, and
$N(-d_1)$ is the Normal Probability function of $-d_1$

$d_1 = [\ln (F_k/R_x) + s^2_T Tt/2)] / s_T v(Tt)$
$d_2 = d_1 - s\, v(Tt)$ and

s is the previously described volatility of 24%

Recognize that the Floorlet is analogous to a Put on the Distribution Amount, and would be "in the money" at any time that the distributable cash was LESS THAN the Strike Price. Graphically then, it can be represented as:



That is, if someone wanted to take a position in each quarterly cash distribution this income trust were expected to pay, and protect themselves against the possibility that any of those distributions were to fall below the Strike Amount, they would purchase a series of Puts that they would exercise sequential in the event that any given distribution fell below the red line. Each individual Floorlet would have a value represented by one Blue valley as identified above. The present value of the entire series of Floorlets, however, is represented by the Black's Model formula stated above.

BOND VALUE LESS FLOORLET IS 'CLASS A' PAYOUT PROFILE

Remember that the payout profile that we are attempting to simulate is one where the Class A Unit Holders will, similar to a 100% equity holder, be subject to all the risks of the physical asset's ability to generate net free cash flow over it's economic life – but only up to a predetermined maximum cash amount in any given quarter [PIR = (CDOR + X%) x $1B]. Otherwise, it is assumed that in any quarter where a lesser amount of cash is generated (i.e. lesser than the strike amount), then all this amount will accrue to the Class A Holders. Referring back to the graph above, it should be recognized that this exact payout profile is represented by the light green area under the blue line, to a maximum of the red line (strike amount)[6].

Also, note that, if this security were a bond, with guaranteed quarterly payments set at the Strike Amount, the present value would simply be all the area under the Red Line (i.e. the nominal amount of each expected constant quarterly distribution discounted back to present day at the risk-free amount). Therefore the Green Area is easily determined by taking the PV of a Bond (under the Red Line) and subtracting the PV of all the Floorlets (as determined by Black's Model). This is the Payout Profile we are attempting to simulate. We can expand the Green Area simply by increasing the X% in our (CDOR + X%) variable, or, conversely, decrease it by reducing the X%. Through a process of

[6] Here it is important to note that, because each quarterly distribution is being modeled individually through a separate discreet iteration of Black's Model (the depiction of the continuous function above represents the continuous floating CDOR rate, but the payment of a quarterly distribution, would obviously be discreet), it is not possible for the valley of the blue line to descend below zero. That is, while it may be possible for net cash flows of the asset to be negative in any given quarter, such a "cash call demand" would not be factored into the overall PV of the Floorlets because both the modeled Strike Amount and all the Risk-Neutral Forward Distributions are positive.

iteration, we can identify the one singe X% that will equate the present value of all the Green Area, throughout the entire economic life of the assets, to $1B. **In the case at hand, we identified that X% to be 9.5% (stated at an annual rate).**

Therefore, in order for the Class A Unit Holder's to be fairly compensated for the maximum ceiling upon the cash distributions that would accrue to their benefit, the appropriate floating rate for those securities to offer is CDOR + 9.5%. Note that this does not infer that the Cost of Capital for this entity is equivalent to this amount – as we had previously identified, the correctly risk-adjusted rate of return that these assets would pay, at the time of acquisition, to a 100% equity holder is a flat 10.2%. However, in order to justify the Class A Unit Holder in foregoing any possible upside that would otherwise be without limit to an equity holder, the Class A should be entitled to a maximum distribution of CDOR + 9.5%. Moreover, it is important to realize that this does not mean the Class A Unit Holder should always expect a CDOR + 9.5% payout. As visually represented in the graphic above, the Class A Holder rides the distribution curve all the way down to zero (just as an equity holder would) in any given quarter where insufficient cash is generated to pay up to the maximum PIR, CDOR + 9.5%.

CONCLUSIONS

Relatively plain derivative models can be quite successfully applied to the application of hybrid securities and in identifying their appropriate rates of return. These models provide a defensible rationale for arriving a reasonable conclusion in a quantified manner whereas any other approach may simply come down the subjective opinions of the valuator.